Sheet (4)
Multiplying and Dividing Rational Numbers

Properties of the Multiplication operation in Q:

(1) Closure property:

The product of any two rational numbers is a rational number.
i.e.: Q is closed under multiplication operation.

(2) Commutative property:

If a and b are two rational numbers, then
$a \times b = b \times a$

(3) Associative property:

If a, b and c are three rational numbers, then
$(a \times b) \times c = a \times (b \times c)$

(4) Multiplicative identity:

One is the multiplicative identity (multiplicative neutral element).
If a is a rational number, then
$1 \times a = a \times 1 = a$

(5) Multiplicative inverse (reciprocal of the number):

For any rational number $\frac{a}{b}$ except zero there is a multiplicative inverse that is the number $\frac{b}{a}$, where: $\frac{a}{b} \times \frac{b}{a} = 1$

- Zero has no multiplicative inverse because $\frac{1}{zero}$ is undefined.
- Multiplying any rational number by zero equals to zero.

(6) Distribution property:

If a, b and c are three rational numbers, then
$a \times (b + c) = a \times b + a \times c$
$a \times (b - c) = a \times b - a \times c$

Properties of operations:

Property \ operation	Addition	Subtraction	Multiplication	Division
Closure	✓	✓	✓	×
Commutative	✓	×	✓	×
Associative	✓	×	✓	×
Identity element	✓ (0)	×	✓ (1)	×
Inverse	✓	×	✓except (0)	×

[1] Complete:

(1) The multiplicative identity element in Q is ....

(2) The multiplicative inverse of $\frac{3}{7}$ is ....

(3) The multiplicative inverse of $\frac{-2}{3}$ is ....

(4) The multiplicative inverse of -6 is ....

(5) The multiplicative inverse of $3\frac{1}{2}$ is ....

(6) The multiplicative inverse of 0.5 is ....

(7) The multiplicative inverse of 1 is ....

(8) The multiplicative inverse of -1 is ....

(9) The multiplicative inverse of $\left(-\frac{3}{5}\right)^{zero}$ is ....

(10) The multiplicative inverse of $\left|-\frac{3}{5}\right|$ is ....

(11) The rational number that has no multiplicative inverse is ....

(12) The rational number $\frac{a-1}{5}$ has a multiplicative inverse if $a \neq$ ......

[2] Put (✓) for the correct statement and (×) for the incorrect one:

(1) Every rational number has a multiplicative inverse. (   )

(2) The multiplicative inverse of a rational number is an integer. (   )

(3) The multiplicative inverse of the number $\frac{0}{7}$ is $\frac{7}{0}$. (   )

(4) The multiplicative inverse of the number $2\frac{1}{5}$ is $5\frac{1}{4}$. (   )

(5) The multiplicative inverse of the number $\left(\frac{2}{7}+\frac{3}{5}\right)$ is $\frac{35}{31}$. (   )

[3] Complete:

The number	The additive inverse	The multiplicative inverse
$\frac{3}{7}$	…………………………	…………………………
$\frac{-4}{9}$	…………………………	…………………………
-6	…………………………	…………………………
0.5	…………………………	…………………………
$3\frac{1}{2}$	…………………………	…………………………
$\left(\frac{-3}{8}\right)^{zero}$	…………………………	…………………………
$\left\lvert -\frac{3}{7} \right\rvert$	…………………………	…………………………
1	…………………………	…………………………
-1	…………………………	…………………………
0	…………………………	…………………………

[4] Complete:

(1) $\frac{3}{2} \times \left(\frac{-4}{5}\right) = \frac{-4}{5} \times \ldots\ldots$ .... property

(2) $\frac{2}{3} \times \frac{3}{2} = \ldots\ldots$ .... property

(3) $7 \times \frac{\ldots}{7} = 1$ .... property

(4) $-\frac{4}{5} \times \ldots\ldots = -\frac{4}{5}$ .... property

(5) $-\frac{4}{11} \times \ldots\ldots = 1$ .... property

(6) $2\frac{3}{5} \times \ldots\ldots = 1$ .... property

(7) $0.8 \times \ldots\ldots = 1$ .... property

(8) $4 \times \ldots\ldots = -5$ .... property

(9) $\frac{2}{3}\left(2 + \frac{1}{2}\right) = \frac{2}{3} \times 2 + \ldots \times \ldots$ .... property

(10) $\frac{3}{9} = \frac{2}{3} \times \frac{\ldots}{8}$

(11) If $\frac{x}{y} = \frac{2}{3}$ then, $\frac{3x}{2y} = \ldots\ldots$

(12) If $\frac{a}{b} = 70$ then $\frac{a}{2b} = \ldots\ldots$

[5] Find out the result of each of the following in the simplest form:

(1) $\frac{3}{5} \times \frac{2}{7} = \ldots\ldots$

(2) $\frac{-1}{2} \times \frac{2}{3} = \ldots\ldots$

(3) $-\frac{3}{8} \times \left(-\frac{5}{3}\right) = \ldots\ldots$

(4) $\frac{2}{6} \times \left(-\frac{3}{4}\right) = \ldots\ldots$

(5) $\left(-\frac{2}{3}\right) \times \frac{5}{8} = \ldots\ldots$

(6) $\frac{4}{5} \times \left(-\frac{5}{7}\right) = \ldots\ldots$

(7) $\left|-\frac{3}{7}\right| \times \left(-\frac{4}{3}\right) = \ldots\ldots$

(8) $\frac{1}{2} \times |-12| = \ldots\ldots$

[6] Find out the result of each of the following in the simplest form:

(1) $\frac{4}{5} \div \frac{3}{7} = \ldots\ldots$

(2) $-\frac{1}{6} \div \frac{5}{2} = \ldots\ldots$

(3) $\frac{-4}{11} \div \left(\frac{-4}{11}\right) = \ldots\ldots$

(4) $\frac{5}{27} \div \frac{1}{9} = \ldots\ldots$

(5) $\frac{5}{6} \div \left(\frac{-15}{2}\right) = \ldots\ldots$

(6) $\frac{-5}{8} \div \frac{5}{8} = \ldots\ldots$

(7) $\text{zero} \div \frac{3}{5} = \ldots\ldots$

(8) $1 \div \frac{7}{5} = \ldots\ldots$

[7] Find out the result of each of the following in the simplest form:

(1) $3\frac{1}{2} \times (-4) = \ldots\ldots$

(2) $1\frac{1}{2} \times \left(\frac{-3}{2}\right) = \ldots\ldots$

(3) $\left(-4\frac{2}{7}\right) \times \left(-5\frac{1}{6}\right) = \ldots\ldots$

(4) $3\frac{1}{8} \times \left(-4\frac{1}{5}\right) = \ldots\ldots$

(5) $\left(-1\frac{1}{2}\right) \times \left|-\frac{5}{3}\right| = \ldots\ldots$

(6) $0.\dot{6} \times 1\frac{1}{3} = \ldots\ldots$

[8] Find out the result of each of the following in the simplest form:

(1) $-2\frac{1}{5} \div \frac{11}{5} = \ldots\ldots$

(2) $-7\frac{5}{6} \div \frac{47}{100} = \ldots\ldots$

(3) $-4\frac{2}{7} \div 1\frac{1}{14} = \ldots\ldots$

(4) $-4\frac{1}{3} \div \left(-3\frac{1}{4}\right) = \ldots\ldots$

(5) $-2\frac{3}{4} \div \left(-3\frac{1}{8}\right) = \ldots\ldots$

(6) $6\frac{1}{4} \div (-15) = \ldots\ldots$

[9] Using the distribution property, find out the result of each of the following in the simplest form:

(1) $\frac{5}{12}\times 3+\frac{5}{12}\times 9=$ ....

(2) $\frac{4}{9}\times 11+\frac{4}{9}\times 16=$ ....

(3) $\frac{6}{37}\times 7+\frac{6}{37}\times 5+\frac{6}{37}\times(-11)=$ ....

(4) $\frac{7}{12}\times 5+\frac{7}{12}\times 9-\frac{7}{12}\times 2=$ ....

(5) $\frac{7}{13}\times 6+\frac{7}{13}\times 8-\frac{7}{13}=$ ....

(6) $\left(\frac{-3}{7}\right)\times 8+5\times\left(\frac{-3}{7}\right)+\left(\frac{-3}{7}\right)=$ ....

[10] Find the result in the simplest form:

(1) $\left(\frac{3}{8}+\frac{5}{8}\right)\div\frac{5}{8}=$ ......

(2) $\frac{3}{4}\times\left(\frac{1}{2}-\frac{1}{3}\right)=$ ......

(3) $\left(\frac{-18}{5}\div\frac{9}{35}\right)\times\left(\frac{-3}{7}\right)=$ ......

(4) $-4\frac{1}{3}\div\left(-3\frac{1}{4}\right)=$ ......

(5) $\left[\frac{-12}{25}\times\left(-\frac{5}{7}\right)\right]\div\left(\frac{-9}{14}\right)=$ ......

(6) $\left[\left(-1\frac{2}{3}\right)\times 4\frac{2}{3}\right]\div 6\frac{1}{9}=$ ......

[11] Find the value of (n) in each of the following:

(1) $\frac{-7}{3}\times\frac{-3}{7}=n$ ....

(2) $n\times\frac{17}{3}=1$ ....

(3) $\frac{-7}{3} \times n = 0$ ....

(4) $\frac{5}{7} \times n = \frac{5}{7}$ ....

(5) $n \times \left[\frac{1}{2} + \left(\frac{-3}{5}\right)\right] = n \times \frac{1}{2} + 5 \times \left(\frac{-3}{5}\right)$ ....

[12] If $a = 2$, $b = \frac{1}{2}$ and $c = \frac{3}{2}$, find in the simplest form the value of:

$(a - b) \div c$

....

[13] If $x = \frac{1}{3}$, $y = \frac{3}{4}$ and $z = -3$, find in the simplest form the numerical value of each of the following:

(1) $x\,y\,z =$ ....

(2) $x\,y + z\,y =$ ....

[14] If $x = \frac{3}{4}$ and $y = \frac{-5}{3}$, find in the simplest form the value of the expression:

$\frac{x - y}{x + y} =$ ....

Sheet (5)
Applications on Rational Numbers

- The distance between two numbers 2 and 5 is:
  $|2-5| = |5-2| = 3$ length units
- The distance between two numbers -2 and 3 is:
  $|-2-3| = |3+2| = 5$ length units
- From the side of the smallest number: $s + f\,(g - s)$
- From the side of the greatest number: $g - f\,(g - s)$

Ex (1): Find the rational number lying at the middle of the way between 3 and 7.

The number = $s + f\,(g - s)$ = ...

Or

The number = $g - f\,(g - s)$ = ...

Ex (2): Find the rational number lying at the half-way between $\frac{2}{5}$ and $\frac{3}{7}$.

The number = $s + f\,(g - s)$ = ...

Ex (3): Find the rational number lying at one third of the way between 2 and 8.

From the side of the smaller number = $s + f\,(g - s)$ = ...

From the side of the greatest number = $g - f\,(g - s)$ = ...

[1] Find the rational number in the middle of the way (half-way) between:

(1) $\frac{3}{8}$ *and* $\frac{5}{8}$ ....

(2) $\frac{-3}{4}$ *and* $\frac{3}{4}$ ....

(3) $\frac{1}{2}$ *and* $\frac{7}{8}$ ....

(4) $\frac{-11}{4}$ *and* $\frac{-13}{35}$ ....

[2] Find the rational number lying at:

(1) One fourth of the way between $\frac{5}{7}$ *and* $\frac{-3}{7}$ from the side of the smaller number.

....

(2) One third of the way between $\frac{-3}{5}$ *and* $\frac{-4}{5}$ from the side of the greater number.

....

(3) One third of the way between $\frac{4}{7}$ *and* $1\frac{3}{4}$ from the side of the smaller number.

....

(4) One fifth of the way between $\frac{-2}{3}$ *and* $\frac{-3}{5}$ from the side of the smaller number.

....

[3] Choose the correct answer:

(1) If $a \times \frac{b}{2} = \frac{a}{2}$, $a \neq 0$, then b = ...

(a) $\frac{a}{2}$ (b) 0 (c) a (d) 1 (e) -a

(2) If $\frac{x}{3} - 4 = 6$, then $\frac{x}{3} + \frac{2}{3}$ = ...

(a) 1 (b) x (c) $\frac{32}{3}$ (d) 10 (e) $\frac{2x}{9}$

(3) If $\frac{x}{y} = 1$, then $2x - 2y$ = ...

(a) 4 (b) 2 (c) 1 (d) 0 (e) $\frac{1}{2}$

(4) If $x + \frac{2}{x} = 5 + \frac{2}{5}$, then x = ...

(a) $\frac{1}{5}$ (b) $\frac{4}{5}$ (c) 1 (d) $\frac{5}{2}$ (e) 5

(5) If $5a = 45$ and $ba = 1$, then b = ...

(a) $\frac{1}{45}$ (b) $\frac{1}{9}$ (c) $\frac{1}{5}$ (d) 5 (e) 9

(6) The number $\frac{x-3}{x-5} \in Q$ if $x \neq$ ......

(a) 3 (b) -3 (c) 5 (d) -5 (e) 15

[4] Find three rational numbers lying between $\frac{3}{2}$ and $\frac{3}{4}$, such that one of them is an integer.

....

....

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Sheet (6)
Algebraic Terms & Algebraic Expressions

The perimeter and the area of some shapes

[1] The square:

- $P = S \times 4 = 4\,S$ (coeff. = 4 and degree = 1st)
- $A = S \times S = S^2$ (coeff. = 1 and degree = 2nd)

[2] Rectangle:

- $P = (\ell + w) \times 2 = 2(\ell + w)$
- $A = \ell \times w = \ell\, w$ (coeff. = 1 and degree = 2nd)

[3] Parallelogram:

- $P = (x + y) \times 2 = 2(x + y)$
- $A = b \times h = b\,h$

[4] Rhombus:

- $P = S \times 4 = 4\,S$
- $A = S \times h = S\,h$ or $A = \frac{1}{2} \times d_1 \times d_2$

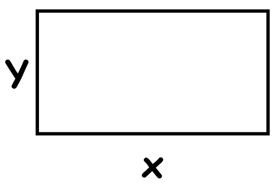
[5] Triangle:

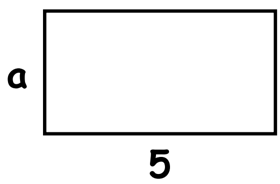
- P = the sum of all side lengths
- Perimeter of equilateral triangle = 3 S
- $A = \frac{1}{2} b\,h$

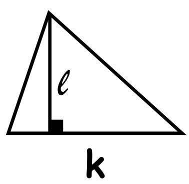
- If we denote one pound by x, if we have 3 pounds
  $x + x + x = 3\,x$ (coeff. = 3 and degree = 1st)

- The algebraic term is formed from the product of two or more factors.
- The degree of the algebraic term is the sum of the indices of the algebraic factors in this term.
- Any number is an algebraic term of zero degree.
- The algebraic term has no algebraic factors is called the absolute term.
- The algebraic expression consists of an algebraic term (monomial) or more.
- The degree of the algebraic expression is the highest degree of its terms.

[1] Write the algebraic term that represent the area of each shape:







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[2] Complete the table:

Algebraic term	$2ab^2$	$7ab^3c$	$-8x^2b$	3	$(-2)^3$	$\frac{1}{2}x^3yz^2$
Coefficient						
Degree						

[3] Complete the table:

The Algebraic expression	No. of terms	Name	Degree
$-3\ a^5\ b$		MONOMIAL	
$3x^2 + y$		BINOMIAL	
$5x^3 - 7x + 4$		TRINOMIAL	
$2a^2\ b + 3a\ b^2 - a^2\ b^2$		TRINOMIAL	
$x^2\ y^2 - 3x\ y^4$		BINOMIAL	
$a^2\ b - 3a\ b^3 + 2a^3\ b^2 + b^4$		QUADRILATERAL	

[4] Complete:

(1) The coefficient of algebraic term $3\ x^2\ y$ is .... and its degree is ....

(2) The coefficient of algebraic term $\frac{1}{2}x^3\ y\ z^2$ is .... and its degree is ....

(3) The degree of the absolute term in an algebraic expression is ....

(4) The algebraic expression $5x^2 + 3$ is of the .... degree.

[5] Choose the correct answer:

(1) The degree of the algebraic term $2x^3y^2$ is ....
(a) second (b) third (c) fourth (d) fifth

(2) The coefficient of the algebraic term $3xy^3z^4$ is ....
(a) 2 (b) 3 (c) 6 (d) 7

(3) The degree of the algebraic expression $3x^2 + 3x^4$ equals to the degree of the algebraic expression ....

(a) $5xy+3y^2z$ (b) $2x^2y^2 + 3x^2y$ (c) $2xy + 3x^4z$ (d) $5a^2b + 4ab^2$

(4) The number of terms of the algebraic expression $3x^2+5xy+6$ is .....

(a) 1 (b) 2 (c) 3 (d) 4

(5) The .... operation is unclosed in the set of rational numbers.

(a) addition (b) subtraction (c) division (d) multiplication

(6) If the degree of the algebraic term $2a^3b^n$ is ninth, then n = ...

(a) 8 (b) 6 (c) 2 (d) 9

(7) The algebraic term b^3 = ...

(a) $3\ b \times b$ (b) $b + b + b$ (c) $b \times b \times b$ (d) $3 \times b$

[6] Arrange the terms of the following algebraic expressions according to the descending order of the indices of a:

(1) $5a + a^2 - 7 + a^3$ = ...

(2) $2\ a^2\ b^2 + 5\ b\ a^3 - 3\ b^3\ a$ = ...

[7] Arrange the terms of the following algebraic expressions according to the ascending order of the indices of x:

(1) $5x + x^2 - 7 + x^3$ = ...

(2) $2\ x^2\ y^2 + 5\ y\ x^3 - 3\ y^3\ x$ = ...

Sheet (7)
Like Algebraic Terms

☞ The algebraic terms are said to be like if they having the same symbols and the same degree. Such as:

Like terms	Unlike terms
☞ $2a$, a and $-5a$. ☞ $2x^2y$, $4yx^2$ and $-\frac{1}{2}x^2y$	☞ $2x$, $-3x^2$ and $7x^3$ ☞ $4x^2$, $5xy$ and y^2

[1] Put (✓) for the correct statement and (×) for the incorrect one:

(1) The two algebraic terms x^2 and $2x$ are like terms. (   )

(2) The two algebraic terms $3\,a\,b^2$ and $-\,a\,b^2$ are like terms. (   )

(3) The two algebraic terms $7x^2$ and $2x^7$ are like terms. (   )

(4) The two algebraic terms $3\,a^2\,b^3$ and $-2\,b^3\,a^2$ are like terms. (   )

(5) $2\,a + 3\,a = 5\,a^2$ (   )

(6) $7\,x^2 - 2\,x^2 = 5\,x^2$ (   )

(7) $8\,y^2 - 5\,y = 3\,y$ (   )

(8) $3\,a\,b - 3\,b\,a =$ zero (   )

[2] Find the result of each of the following:

(1) $3\,x + x =$ ....

(2) $7\,y - y =$ ....

(3) $3\,x + 2\,x =$ ....

(4) $5\,y - 3\,y =$ ....

(5) $4\,z - 11\,z =$ ....

(6) $-\,7\,x - 2\,x =$ ....

(7) $2\,a + 3\,a - 4\,a =$ ....

(8) $-\,3\,a^2 + 5\,a^2 =$ ....

(9) $\frac{5x}{4} + \frac{3x}{4} =$ ....

(10) $\frac{3x}{5} - \frac{x}{5} =$ ....

[3] Answer each of the following:

(1) Subtract y^2 from $-3y^2$ ....

(2) Subtract $-6x^2y$ from $9x^2y$ ....

(3) What is the increase $-2x$ of $-5x$? ....

(4) What is the increase $3a^2b$ of a^2b? ....

(5) What is the decrease $-3ab$ of $2ab$? ....

(6) What is the decrease $6x^2y$ of $-7x^2y$? ....

[4] Complete:

(1) The result of subtracting $3a$ from $7a$ is ....

(2) The result of subtracting $3x^2$ from $-5x^2$ is ....

(3) The result of subtracting $7y^3$ from zero is ....

(4) The result of subtracting $-3a$ from $2a$ is ....

(5) $5a$ increases $3a$ by ....

(6) $7x$ increases $-3x$ by ....

(7) $4x$ decreases $7x$ by ....

(8) $5x$ decreases $3x$ by ....

(9) $2x$ decreases $4x$ by ....while $2x$ increases $4x$ by ....

(10) .... $+ 2a^2 = 7a^2$

(11) $3x^2 -$ .... $= x^2$

(12) $2m^2 +$ .... = zero

(13) $5\ a^2\ b -$ .... $= 7\ a^2\ b$

(14) If $4x - y = 11$ and $y = 3x$, then $x =$ ....

[5] If the sum of two terms is 12 x2 y one of them is 4 x2 y. Find the other term.

....

[6] Reduce to the simplest form:

(1) $3a + 2b + 5a + 4b =$ ....

(2) $2x - 4y - 9x - 3y =$ ....

(3) $3x - 5y - x + 2y =$ ....

(4) $19\,m - 4\,n + 11\,m - 17\,n + 9\,n =$ ....

(5) $4\,a + a\,b + 5\,a - 2\,b + 6\,b - 3\,a =$ ....

[7] Reduce each of the following algebraic expressions:

(1) $5x + 4 - 3x^2 - 6x - 7x^2 - 1 =$ ....

....

....

(2) $6\,x^2\,y - 3\,x\,y^2 + 2\,x\,y^2 - 5\,x^2\,y + 2\,x^2\,y^2 =$ ....

....

....

(3) $a^2 + 4a - 5 + 3a^2 - 6a + 1 =$ ....

....

....

(4) $5\,x^2 - 2\,x + 8 - 7\,x - 3 + x^2 =$ ....

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Sheet (8)
Adding and Subtracting Expressions

[1] Find the sum of each of the following:

(1) $3x - 2y + 5$ and $x + 2y - 2$

(2) $3n^2 + 5n - 6$ and $-n^2 - 3n + 3$

(3) $3\ell - 4m + 5n$ and $4m - 5n - \ell$

(4) $3a^3 - 2a^2b + b^3$ and $a^3 + 4a^2b - b^3$

[2] Find the sum:

(1) $3a + 2b - 5$, $2a - 7b + 4$, $5b - 4a + 3$

(2) $3x + 3y - z$, $3x + 3z - 2y$, $x + 2y + z$

....

....

....

....

....

(3) $5x^2 - 3x + 9$, $x^2 + 2x - 5$, $x^2 - 3 - 6x$

....

....

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....

....

(4) $3x - 4x^2 + 2$, $x^2 + x - 5$, $3 + 3x^2 - 4x$

....

....

....

....

....

[3] Subtract:

(1) $x - 2$ from $2x - 5$

(2) $2x + 6y - 7$ from $2x - 5y + 2$

[4] What is the increase of:

(1) $5a + 7b$ than $3a - 2b$

(2) $x^2 - 5x - 1$ than $3x^2 + 2x - 3$

[5] What is the decrease of:

(1) $2a + 3b$ than $5b - 3a$

(2) $3y^2 - 2xy + x^2$ than $3x^2 - 5xy + y^2$

[6] Subtract x + x2 – 5 from 2x2 + x – 3, then find the numerical value of the result when x = 6

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Sheet (9)
Multiplying and Dividing Algebraic Terms

[1] Multiply:

(1) $5x \times 3y$ = ...

(2) $(-3a) \times 7c$ = ...

(3) $2x \times (-3x)$ = ...

(4) $(-8y^5) \times (-7y^4)$ = ...

(5) $2xy \times (-3x^2)$ = ...

(6) $5x^3y^4 \times 2xy^2$ = ...

(7) $5ab^2 \times (-2a^2b)$ = ...

(8) $ab \times (-3a) \times (-2b)$ = ...

(9) $2x^3 \times (-3x^2) \times (-5x^4)$ = ...

(10) $(-2x) \times 4x$ = ...

[2] If the symbols represent non-zero integers, find the quotient of each of the following:

(1) $6a \div 2$ = ...

(2) $10c \div 2c$ = ...

(3) $12x \div (-x)$ = ...

(4) $(-14x^2) \div 7x$ = ...

(5) $\left(-25a^6\right) \div \left(-5a^2\right)$ = ...

(6) $24c^5 \div \left(-24c^5\right)$ = ...

(7) $9x^5y^4 \div 6x^3y$ = ...

(8) $\left(-32a^3b^6\right) \div \left(-4a^3b^2\right)$ = ...

(9) $8m^4n^3 \div \left(-4m\,n^2\right)$ = ...

[3] Simplify:

(1) $\frac{2}{3}t^4 \times \frac{3}{2}t^4$ = ...

(2) $\frac{2}{7}a^2 \times 21a^5$ = ...

(3) $\frac{6x^4y^2}{7} \times \frac{28x\,y^3}{3}$ = ...

(4) $3x^3 \times \frac{1}{6}x^2$ = ...

[4] Choose the correct answer:

(1) $3a^4b \times 5a^2b^2 \times 2a^3$ = ...

(a) $60a^{11}b^3$ (b) $30a^{10}b^2$ (c) $150a^{10}b^3$ (d) $30a^9b^3$

(2) $\left(-3x^2y\right)^2 \times 2xy$ = ...

(a) $-18x^5y^3$ (b) $18x^5y^3$ (c) $6x^3y^2$ (d) $9x^2y^2$

(3) $\left(-6x^3y^2\right) \div 3x^2y$ = ...

(a) $-2x^2y$ (b) $2xy$ (c) $-2xy$ (d) $-2x^2y^2$

(4) If $2b$ cm is the edge length of a cube, then its volume = ... cm^3

(a) $4b^2$ (b) $2b^3$ (c) $4b^3$ (d) $8b^3$

(5) If the area of a rectangle is $24x^3$ cm^2 and its length is $8x^2$ cm, then its width is ....

(a) $3x$ (b) $3x^2$ (c) $4x$ (d) $4x^5$

[5] Complete:

(1) $9a^5 = 3a \times \ldots\ldots$

(2) $36a^5b^8 = 12a^3b^2 \times \ldots\ldots$

(3) $-4c^3d^3 = 2c\,d^2 \times \ldots\ldots$

(4) $81l^4 \div \ldots\ldots = 27l^3$

(5) $\ldots\ldots \div 6a^2 = -4a^4$

(6) $36a^7b^4 = \ldots\ldots \times 9a^7b$

Sheet (10)
Multiplying a monomial by an algebraic expression

[1] Find the following products:

(1) $a(a+1)$ = ……………………………

(2) $a(a-2)$ = ……………………………

(3) $3x(7y-4z)$ = ……………………………

(4) $-3(y+3)$ = ……………………………

(5) $-2c(7-3c)$ = ……………………………

(6) $2x(3x^2+4y^2)$ = ……………………………

(7) $-5x(2x+y-3z)$ = ……………………………

(8) $3xy(2x^2-5x^2y-4y^2)$ = ……………………………

(9) $l\,m^2(l^2-3m\,l-4m^2)$ = ……………………………

(10) $\frac{1}{3}x^2(6x^2-9x\,y-3y^2)$ = ……………………………

[2] Put in the simplest form:

(1) $3a(a-b)+4a(2a+b)$

= ……………………………………………………………………………

= ……………………………………………………………………………

(2) $3a(4a-2)-4a(3a-2)$

= ....

= ....

[3] Simplify $2a(3a-1)+3a(a+2)$, then find the numerical value of the result when a = 1:

$2a(3a-1)+3a(a+2)$

= ....

= ....

= ....

= ....

Sheet (11)
Multiplying a binomial by an algebraic expression

We have 3 ideas of the examples on this lesson

1st idea this is the general idea

[1] Find by direct products:

(1) $(x+3)(x+2)$ = ...

(2) $(x-3)(x-2)$ = ...

(3) $(x+2)(x-5)$ = ...

(4) $(y-4)(y+5)$ = ...

(5) $(x+2)(x+4)$ = ...

(6) $(y-5)(y+2)$ = ...

(7) $(5m-2)(6m+1)$ = ...

(8) $(4x+1)(2x+3)$ = ...

(9) $(3a+2b)(2a-5b)$ = ...

(10) $(b^2-4)(b^2+2)$ = ...

(11) $(x-y)(7y-x)$ = ...

2nd idea (special case of 1st idea)

[2] Find by inspection the expansion of each of the following:

(1) $(x+2)^2$ = ...

(2) $(x+3)^2$ = ...

(3) $(x+1)^2$ = ...

(4) $(x-1)^2$ = ...

(5) $(2y+3)^2$ = ...

(6) $(4m-7)^2$ = ...

(7) $(3x+y)^2$ = ...

(8) $(x-3y)^2$ = ...

(9) $(2x+3y)^2$ = ...

(10) $(-l-m)^2$ = ...

(11) $(-4x-7)^2$ = ...

3rd idea special case of 1st idea

[3] Find by inspection the expansion of each of the following:

(1) $(x+3)(x-3)$ = ...

(2) $(x-4)(x+4)$ = ...

(3) $(x-2)(x+2)$ = ...

(4) $(4m-7)(4m+7)$ = ...

(5) $(6x+2y)(6x-2y)$ = ...

(6) $(a^2+a)(a^2-a)$ = ...

(7) $(3x^2+5y^2)(3x^2-5y^2)$ = ...

(8) $\left(\frac{1}{2}x+\frac{1}{3}y\right)\left(\frac{1}{2}x-\frac{1}{3}y\right)$ = ...

[4] Choose the correct answer:

(1) The middle term in the expansion of $(3x-1)^2$ is ....

(a) $3x$ (b) $-6x$ (c) $6x$ (d) $6x^2$

(2) The middle term in the expansion of $(2a+3b)^2$ is ....

(a) $12ab$ (b) $-12ab$ (c) $6ab$ (d) $-6ab$

(3) If $(2x+y)^2 = 4x^2 + kxy + y^2$, then $k = \ldots\ldots$

(a) 2 (b) 4 (c) 8 (d) 6

(4) If $x=-1$, then the numerical value of $(x+1)^2$ is ....

(a) zero (b) 1 (c) 2 (d) 3

(5) If $x^2 = 16$, $y^2 = 9$ and $xy = 12$, then $(x-y)^2 = \ldots\ldots$

(a) 49 (b) 165 (c) -1 (d) 1

(6) If $(x+y)^2 = 26$ and $x^2+y^2 = 20$, then $xy = \ldots\ldots$

(a) 3 (b) 6 (c) 9 (d) 12

(7) If $x + y = 7$, then the numerical value of $x^2 + 2xy + y^2 = \ldots$

(a) 7 (b) 14 (c) 49 (d) 28

(8) If $x - y = 3$ and $x + y = 5$, then $x^2 - y^2 = \ldots\ldots$

(a) 2 (b) -2 (c) 8 (d) 15

(9) If $x = \frac{4}{3}$, then $(x - 2)(x + 2) = \ldots\ldots$

(a) $\frac{4}{3} - 2$ (b) $\left(\frac{4}{3}\right)^2 - 2$ (c) $\left(\frac{4}{3}\right)^2 - 4$ (d) $\left(\frac{4}{3}\right)^2 + 4$

(10) If $(x - 3)(x + 3) = x^2 + k$, then $k = \ldots\ldots$

(a) 9 (b) 6 (c) -9 (d) -6

(11) If $(x - y)(2x + y) = 2x^2 + k\,x\,y - y^2$, then $|k| = \ldots\ldots$

(a) -1 (b) 1 (c) 3 (d) 4

[5] Multiply, then find the numerical value of the expression when $x = 1$ and $y = -2$:

(1) $(x - 5y)(x + 5y)$ = ...

= ....

(2) $(3x + y)(x + 3y)$ = ...

= ....

(3) $(x + 4)(3x + 2)$ = ...

= ....

[6] Reduce $(x - y)^2 + 2\,x\,y$, then find the numerical value of the result when $x = -1$ and $y = -2$:

= ....

= ....

= ....

= ....

[7] Reduce = $(2x - 2)^2 + (x - 2)(x + 2)$, then find the numerical value of the result when $x = -1$:

= ....

= ....

= ....

= ....

[8] Simplify to the simplest form $(2a - 3)(2a + 3) + 7$, then find the numerical value of the result when $a = -1$:

= ....

= ....

= ....

= ....

Sheet (12)

Dividing an algebraic expression by a monomial

[1] If the symbols in the following expressions are non-zero numbers, find the quotient in each case:

(1) $5a - 10$ by 5 = ...

(2) $4a^2 + 6a$ by $2a$ = ...

(3) $12a^2b + 20a\ b^2$ by $4a\ b$ = ...

(4) $16a^3b^2 - 24a^2b^2$ by $4a^2b$ = ...

(5) $12x + 15y$ by -3 = ...

(6) $24x^3 - 18x^2$ by $-6x^2$ = ...

(7) $60x^6 - 48x^{10} - 12x^3$ by $-12x^3$ = ...

= ....

(8) $32x^5 - 48x^3 + 72x^7$ by $-8x^3$ = ...

= ....

[2] Find the quotient of each of the following:

(1) $\dfrac{26x^2 + 14x^4}{2x}$ = ... = ...

(2) $\dfrac{18m^4 + 32m^2}{-2m^2}$ = ... = ...

(3) $\dfrac{48x^3 - 80x^2}{8x^2}$ = ... = ...

(4) $\dfrac{9l^3m^4 - 18l\ m^2}{3l\ m^2}$ = ... = ...

[3] Choose the correct answer:

(1) $(x^2 + x) \div x = \ldots\ldots, \quad x \neq 0$

(a) zero (b) x (c) $2x + 1$ (d) $x + 1$

(2) $(15a + 5) \div 5 = \ldots\ldots$

(a) $3a$ (b) $10a$ (c) $3a + 1$ (d) $4a$

(3) $(4a^3 - 2a) \div (-2a) = \ldots\ldots, \quad a \neq 0$

(a) $-2a^2$ (b) $-2a^2 + 1$ (c) $2a^2 + 1$ (d) -1

(4) $(15x^4 + 5x^3) \div 5x^3 = \ldots\ldots$

(a) $3x^2 + x$ (b) $5x^2 + 1$ (c) $3x + 1$ (d) $4x^4$

(5) $(3x^2y - \ldots\ldots) \div 3x\ y = x - 2y$

(a) $6x$ (b) $6x\ y^2$ (c) $6y^2$ (d) $-6x\ y^2$

(6) If $(6x^2y^3 + k\ x\ y) \div 6x = x\ y^3 - 12y, \ x \neq 0$, then $|k| = \ldots\ldots$

(a) -72 (b) -2 (c) 2 (d) 72

Sheet (13)

Dividing an algebraic expression by another one

[1] Find the quotient of each of the following:

(1) $x^2 + 5x + 6$ by $x + 2$

(2) $y^2 - 9y + 20$ by $y - 4$

(3) $x^2 - 5x - 14$ by $x - 7$

(4) $2x^2 + 13x + 15$ by $x + 5$

(5) $3x^2 + 2x - 8$ by $3x - 4$

(6) $x^2 - 6 - x$ by $x + 2$

[2] If the area of a rectangle is $(15x^2 + 11x - 14)$ cm2 and its width is $(3x - 2)$ cm. Calculate its length.

[3] If the area of a rectangle is $(2x^2 + 7x - 15)$ cm2 and its length is $(x + 5)$ cm. Find its width and calculate its perimeter when $x = 3$.

1-4 Multiplying and dividing rational numbers

EXAMPLE: **Find the result of each of the following in its simplest form :**

1) $\frac{3}{6} \times \frac{2}{5}$

2) $-\frac{3}{4} \times \frac{2}{9}$

3) $\frac{1}{2} \times (-2)$

4) $-4\frac{2}{7} \times (-3\frac{1}{6})$

5) $\frac{3}{2} \times \frac{5}{9}$

6) $\frac{8}{5} \times (-\frac{4}{9})$

7) $-5 \times \frac{3}{10}$

8) $-4\frac{1}{2} \times (-\frac{5}{9})$

Properties of the set of rational numbers under multiplication

1 Closure property :

2 Commutative property :

3 Associative property :

4 The existence of multiplicative identity (neutral) element property :

....

....

5 The existence of multiplicative inverse of the rational number property :

....

....

6 Property of distributing multiplication over addition and subtraction :

....

....

Example: **Use the distributive property to find each of the following :**

9) $\frac{5}{11} \times \frac{6}{7} + \frac{5}{11} \times \frac{1}{7}$

....

....

....

....

10) $\frac{9}{17} \times 21 - \frac{9}{17} \times 4$

11) $\frac{22}{25} \times \frac{6}{11} + \frac{5}{11} \times \frac{22}{25} - \frac{22}{25}$

12) $\frac{7}{12} \times 5 + \frac{49}{12} - \frac{7}{12} \times 11$

13) $\frac{5}{7} \times \frac{2}{3} + \frac{5}{7} \times \frac{1}{3}$

14) $11 \times \frac{3}{10} - \frac{3}{10}$

Example: **Find the result of each of the following in its simplest form :**

15) $-\frac{2}{3} \div \frac{5}{3}$

16) $\frac{3}{7} \div (-8)$

17) $2\frac{1}{5} \div 5\frac{1}{2}$

18) $0.2 \div \frac{1}{5}$

19) $(\frac{2}{7} + \frac{3}{7}) \div \frac{10}{7}$

20) $(\frac{5}{6} - \frac{3}{4}) \div (\frac{7}{12} - \frac{5}{9})$

21) $\frac{3}{7} \div \frac{9}{14}$

22) $\frac{3}{4} \div (-\frac{15}{2})$

23) $2\frac{1}{3} \div (-\frac{7}{3})$

24) $-\frac{5}{6} \div 10$

Example:

If $x = -\frac{1}{3}, y = \frac{3}{4}$ and $z = -3$, find the numerical value of each of the following :

25) $\frac{y}{z}$

26) $\frac{xy}{z}$

27) $\frac{x}{y} - \frac{y}{z}$

1-5 applications on the rational numbers

Example:

1) Find a rational number lying at one third of the way between 2 and 8

2) Find a rational number in the half way way between $\frac{2}{5}$ and $\frac{3}{7}$

Example: Find a rational number lying at one third of the way between 2 and 8

3) **From the side of the smaller number.**

4) **From the side of the greater number.**

Example:

5) Find a rational number lying at one fourth of the way way between $-\frac{1}{6}$ and $-\frac{1}{3}$ from the side of the smaller nummber

....

....

....

6) **Find a rational number lying at one fifth of the way between : $\frac{2}{5}$ and $\frac{4}{7}$ from the side of the greater number.**

....

....

....

2-1 Algebraic terms and algebraic expressions

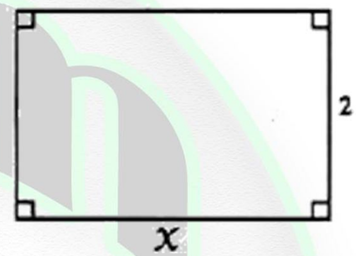
Example:

Write the algebraic term that represents the area of each of the following shapes :

1) ....

....

....



2) ....

....

....

Example: **Complete the following table :**

3)

Algebraic term	$5x$	$3xy$	$-5a^2$	$4x^2y$	$-2a^2b^2$	$15a^3b$	x	-4	$(-3)^2$
Its coefficient									
Its degree									

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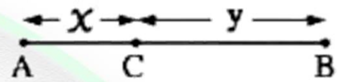
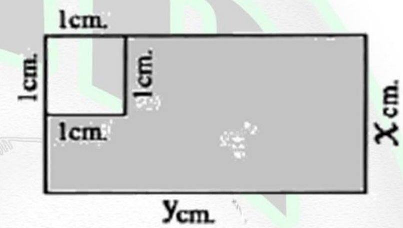
EXAMPLE: Write the algebraic expression that expresses each of the following :

4) The length of $\overline{AB}$



5) The area of the shaded part



EXAMPLE: Arrange the algebraic expression : $5x + 2x^3 - 4 - x^2$:

6) According to the descending order of the indices of x

7) According to the ascending order of the indices of x

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EXAMPLE:

State the degree of the algebraic expression : $2\,a^3\,b^2 - 7\,a\,b^3 + 5\,a^2\,b$, then arrange it :

8) According to the descending order of the indices of a

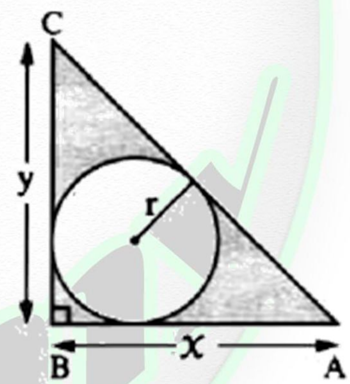
....

9) According to the ascending order of the indices of b

....

EXAMPLE:

10) **From the opposite figure :**
Write the algebraic expression which represents the area of the coloured part , then state its degree (given that the area of the circle = $\pi\,r^2$)



....

....

....

....

....

....

....

Example:

Complete the following table :

11)

The algebraic expression	Its terms	The number of terms	Its name	Its degree
$-2a^2b^3$	$-2a^2b^3$	1	Monomial	5
$a^3-5a^2b^2+3b^2$	$a^3, -5a^2b^2, 3b^2$	....	Trinomial	....
$\frac{1}{2}a+\frac{1}{4}b-5$	....	....	....	....
$2x^2y+5xy+4y$	....	....	....	....
$1-7x^2y$	....	....	....	....
$3^2x^2+2^4x$	....	....	....	....

2-2 like algebraic terms

Example: Add :

1) $5a, 3a, a, 6a$

....

2) $7ab^2, -2b^2a, -4b^2a, ab^2$

....

Example: Subtract :

3) $5xy$ from $7xy$

....

....

4) $2x^2y$ from $-5x^2y$

....

....

5) $-3a^2b^2$ from $5a^2b^2$

....

....

6) $-3x^3y$ from $-2yx^3$

....

....

EXAMPLE: **Put the suitable term in each space :**

7) $4x + 5x = \boxed{}$

....

8) $2x - 4x + x = \boxed{}$

....

9) $3x^2 + \boxed{} = 5x^2$

....

10) $7a^3 - \boxed{} = 2a^3$

....

11) $2b^4 + \boxed{} = b^4$

....

12) $3y^5 - \boxed{} = 5y^5$

....

13) $4x$ is less than $7x$ by $\boxed{}$

14) 7 y is more than – 2 y by $\boxed{}$

Example: **Reduce to the simplest form :**

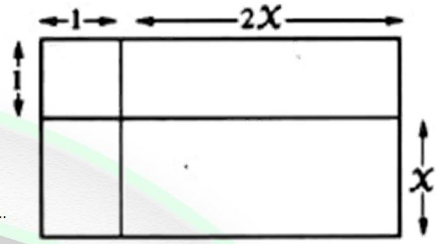
15) $6x + 7y + 4x - 3y$

16) $6x^2 - 7x - 4x^2 + 5x - 3x + x^2$

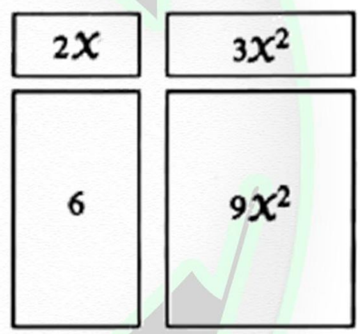
17) $a^2 + 3a - 4 + 4a^2 - 5a + 1$

Example:

18) **In the opposite figure :**
Write the algebraic expression that
expresses the perimeter of the opposite figure.



19) **In the opposite figure :**
Write the algebraic expression which
expresses the sum of the areas of the rectangles which are
shown in the opposite figure.



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2-3 Adding and subtracting algebraic expressions

Example: **Add the following expressions :**

1) $5a - 7b + 3$ and $2b - 1 - a$

2) $3x^3 - 4x^2 + 2x - 1$, $5x^2 - 2x^3 + 3$ and $2 - 3x + x^2$

Example:

3) **Add :** $4x^2 - 3xy + y^2$ **and** $3xy - 3x^2 + 2y^2$

Then find the numerical value of the result when : $x = -2$, $y = 1$

4) **Add :** $3x^2 - 5 + 2x$, $x + 5x^2 + 7$ **and** $-4x^2 - 3$

Then find the numerical value of the result when : $x = 2$

Example: **Subtract :**

5) $5x - 3y + 2z$ from $2y - z + 7x$

Example:

6) **What is the expression that should be added to $8 - 3a^2 + 2a^3$ to get the result $5 + 4a^3 - 7a$?**

7) **What is the increase of $3a^2 - 4b^2 + 2ab$ than the sum of $2a^2 - 3ab + b^2$ and $2b^2 + a^2 + ab$?**

8) What is the decrease of $7 - 5a + a^2$ than $3a^2 - 5a - 2$?

2-4 Multiplying and dividing algebraic terms

EXAMPLE: **Find the result of the following :**

1) $5\,a^3 b \times 3\,ab$

2) $\frac{3}{4}\,a^2 \times \frac{4}{3}\,a$

3) $\frac{2}{5}\,x^2 \times (-15\,x^3)$

4) $2\,a \times (-3\,ab)$

5) $-2\,x^2 y \times 3\,x y^2$

6) $\frac{2}{3}\,m^2 n \times \frac{9}{4}\,n$

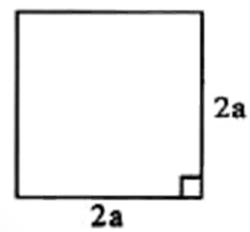
7) $-4\,\ell m^2 \times \frac{1}{2}\,\ell^2 m^2$

EXAMPLE: **Calculate the perimeter and the area of each figure of the following :**

8) ....

....

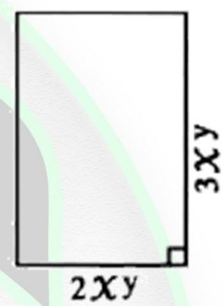
....



9) ....

....

....

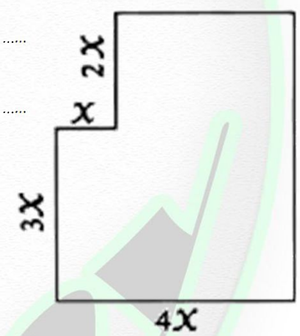


10) ....

....

....

....



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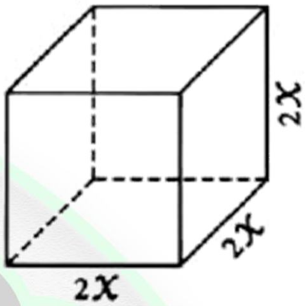
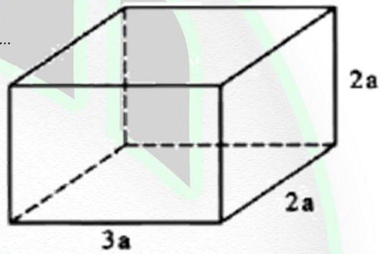
EXAMPLE: **Calculate the total surface area and the volume of each solid of the following :**

11) ....

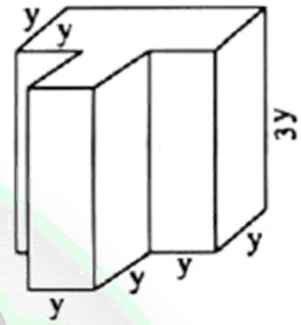


12) ....

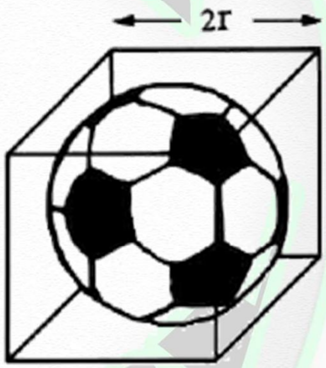


Example:

13) **Find the volume of the opposite solid.**



14) A sphere is put inside a cube as shown in the figure to touch all its six faces internally.
Find the ratio between the volume of the sphere and that of the cube $(\pi = \frac{22}{7})$.



(The volume of the sphere = $\frac{4}{3}\pi r^3$)

2-5 Multiplying a monomial by an algebraic expression

Example: **Find the product of each of the following :**

1) $b(-2a + a^2 b)$

....

2) $-3ab(5a - 2b + 3)$

....

3) $(a^2 - ab - 2b^2) \times 4ab$

....

4) $3a\ (2a - 4b)$

....

5) $-2x(3xy - 5x)$

....

Example:

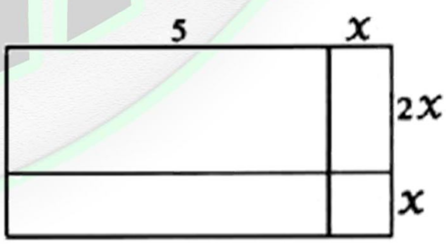
6) **Simplify to the simplest form :** $2a(a + 4b) - 3b(a - 3b) - (2a^2 + 8b^2)$

Then find the numerical value of the result when : $a = 1$ **and** $b = -2$

7) **Reduce the following to the simplest form :** $2x(3x - 2) + 3x(x + 1)$

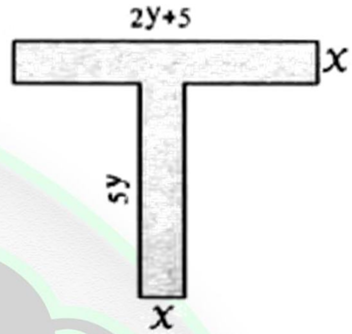
Example: **Find the area**

8)

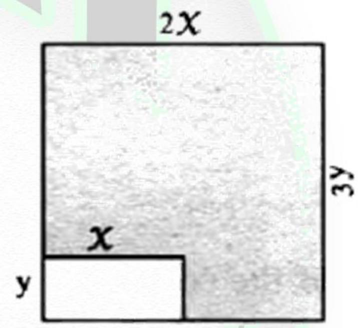


EXAMPLE: **Find the area of the coloured part in each of the following figures :**

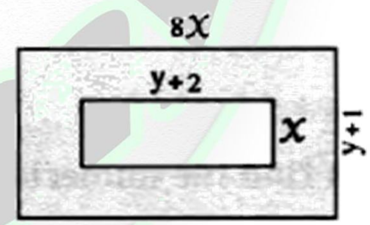
9)



10)



11)



2-6 Multiplying a binomial by an algebraic expression

EXAMPLE: **Find the product of :**

1) $(x+5)(2x-3)$

2) $(3x+7)(2x-3)$

3) $(2a+3)(5a+1)$

4) $(3x+4)(2x-5)$

5) $(5a-2b)(7a-3b)$

6) $(4x-3y)(3y+x)$

7) $(2a+1)(5a+3)$

8) $(3x+4)(2x-1)$

Example: **Find the expansion of each of the following :**

9) $(3a+5)^2$

10) $(2x-3y)^2$

11) $(3m+2)^2$

12) $(2x-3y)^2$

Example: **Find the product of each of the following :**

13) $(2\ell-5)(2\ell+5)$

14) $(5x+3y)(5x-3y)$

15) $(a^2+2b)(a^2-2b)$

16) $\left(\frac{1}{3}a-\frac{2}{5}b\right)\left(\frac{1}{3}a+\frac{2}{5}b\right)$

17) $(2a+3b)(2a-3b)$

18) $(3a-4b)(3a+4b)$

Example: **Put each of the following in the simplest form :**

19) $(x+4)^2-(x+2)(x+6)$

20) $(x+5)(x-5)+(x-5)^2$

Example: **Find the product of :**

21) $(x-3)(x^2+4x-7)$

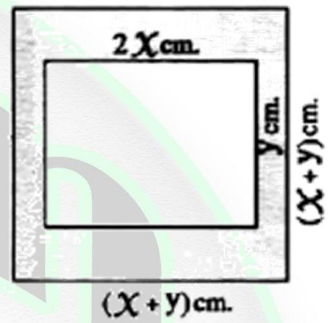
22) $(-3x+x^2+3)(x-2)$

23) $3a^3+a^2-4$ **by** $2a+3$

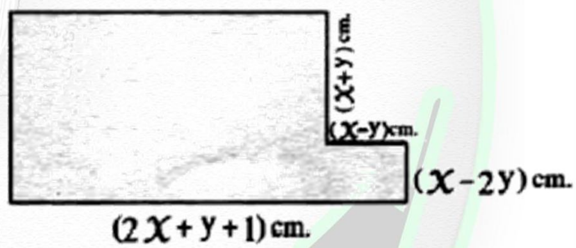
EXAMPLE:

Find the expression which expresses the area of the coloured part in each of the following figures :

24) ....



25) ....



EXAMPLE:

Use the multiplication by inspection to find the value of each of the following easily the value of :

26) $(52)^2$

27) $(195)^2$

28) 502×498

2-7 Dividing an algebraic expression by a monomial

Example:

Find the quotient in each of the following where the symbols represent integers which aren't equal to zero :

1) $\dfrac{21x^2 + 14x}{7x}$

2) $\dfrac{10a^6b^4 - 8a^5b^3 + 2a^4b^2}{2a^4b}$

3) $(16x^3y + 8x^2y^3 - 12x^2y)$ by $(-4x^2y)$

4) $(12x^4 + 8x^2) \div 4x$

5) $(14x^3 - 21x^2 + 7x) \div (-7x)$

EXAMPLE:

6) **Divide :** $\frac{3\,a\,b^2\,c - 5\,a^2\,b\,c + 2\,a\,b\,c^2}{a\,b\,c}$ in which abc ≠ zero

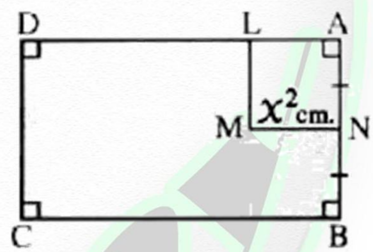
, then find the absolute value of the result when: a = 1 , b = – 2 and c = 3

EXAMPLE:

7) **In the opposite figure :**

ABCD is a rectangle , ANML is a square , N is the midpoint of $\overline{AB}$ and NM = x^2 cm. If the area of the coloured part is $(x^4 + 10\,x^2)$ cm^2.

Find the length of $\overline{LD}$



2-8 Dividing an algebraic expression by a another one

Example: **Find the quotient of dividing :**

1) $5a - 10a^2 + 6a^3 + 3$ by $3 + 2a^2 - 4a$

2) $x^3 + x + 10$ **by** $x + 2$ where $x \neq -2$

3) If $(x - 1)$ is one of the factors of $(x^2 + 5x - 6)$, then find the other factor.

4) **If the expression** $(2x^3 + 11x^2 + 12x + m)$ is divisible by $(x + 3)$, find the value of m

a If $x = -\frac{1}{3}$, $y = \frac{3}{4}$ and $z = -3$, **find the numerical value of :** $4xy + z$

Solution $= 4 \times \frac{-1}{3} \times \frac{3}{4} + (-3) = -1 - 3 = -4$

b If $x = \frac{3}{4}$, $y = -\frac{5}{2}$, **find in simplest form the value of :**

$(x - y) \div (x + y)$ Solution $= \left(\frac{3}{4} - \left(\frac{-5}{2}\right)\right) \div \left(\frac{3}{4} + \left(\frac{-5}{2}\right)\right)$

$= \left(\frac{3}{4} + \frac{10}{4}\right) \div \left(\frac{3}{4} - \frac{10}{4}\right)$

$= \frac{13}{4} \div \frac{-7}{4} = \frac{13}{4} \times \frac{-4}{7} = \frac{-13}{7}$

c If $x = \frac{1}{2}$, $y = \frac{-2}{3}$, $z = 2$, **find the value of :** $\frac{y - z}{x}$

Solution $y - z = \frac{-2}{3} - 2 = \frac{-2}{3} - \frac{6}{3} = \frac{-8}{3} \div \frac{1}{2} = \frac{-8}{3} \times 2 = \frac{-16}{3}$

d If $x = \frac{2}{3}$, $y = \frac{-3}{4}$, $z = -3$, **find the value of :** $xy - z$

Solution $\frac{\cancel{2}^{1}}{\cancel{3}} \times \left(\frac{\cancel{-3}}{\cancel{4}_{2}}\right) - (-3) = \frac{-1}{2} + 3 = 2\frac{1}{2}$

E If $a = \frac{1}{2}$, $b = -\frac{2}{3}$ and $c = 3$ **Find the value of :** $a^2 - 2bc$

Solution $= \left(\frac{1}{2}\right)^2 - 2 \times \left(\frac{-2}{\cancel{3}}\right) \times \cancel{3} = \frac{1}{4} + 4 = 4\frac{1}{4}$

F If $a = \frac{7}{4}$, $b = \frac{-1}{2}$, **find the value of :** $(a - b) \div (a + b)$

Solution $= \left(\frac{7}{4} - \left(\frac{-1}{2}\right)\right) \div \left(\frac{7}{4} + \left(\frac{-1}{2}\right)\right) = \left(\frac{7}{4} + \frac{2}{4}\right) \div \left(\frac{7}{4} - \frac{2}{4}\right)$

$= \frac{9}{4} \div \frac{5}{4} = \frac{9}{4} \times \frac{4}{5} = \frac{9}{5}$

A

If $x = \frac{3}{2}$, $y = -\frac{1}{4}$ and $z = -2$,

find in the simplest form the numerical value of the following : $x - (z \div y)$

Solution $= \frac{3}{2} - [-2 \div (-\frac{1}{4})]$

$= \frac{3}{2} - [-2 \times (-4)]$ $\quad = \frac{3}{2} - 8 = -\frac{13}{2}$

B

If the two rational numbers $\frac{3x}{4}$ and $\frac{2}{3}$ are equal , **find the value of** x

Solution $\frac{3x}{4} = \frac{2}{3}$ $\quad x = \frac{4 \times 2}{3 \times 3} = \frac{8}{9}$

C

If $a = \frac{3}{4}$, $b = -\frac{5}{2}$, without using calculator find the value of : $4a - 6b$

Solution The numerical value $= 4 \times \frac{3}{4} - 6 \times \frac{-5}{2} = 3 + 15 = 18$

D

Find in the simplest form the value of each of the following :

① $-15\frac{1}{4} + 12\frac{1}{2}$

Solution $-15\frac{1}{4} = -\frac{61}{4}$, $12\frac{1}{2} = \frac{25}{2}$

$-15\frac{1}{4} + 12\frac{1}{2} = \frac{-61}{4} + \frac{50}{4} = -\frac{11}{4} = -2\frac{3}{4}$

② $0.\dot{1}\dot{8} - 25\ \%$

Solution

$0.\dot{1}\dot{8} = \frac{2}{11}$, $25\ \% = \frac{1}{4}$

$\frac{2}{11} - \frac{1}{4} = \frac{8}{44} - \frac{11}{44} = -\frac{3}{44}$

F

If $a = \frac{7}{4}$, $b = \frac{1}{2}$, find the numerical value of the expression : $\frac{a-b}{a+b}$

$a - b = \frac{7}{4} - \frac{1}{2} = \frac{7}{4} - \frac{2}{4} = \frac{5}{4}$

$a + b = \frac{7}{4} + \frac{1}{2} = \frac{7}{4} + \frac{2}{4}$

$\frac{a-b}{a+b} = \frac{5}{4} \times \frac{4}{9} = \frac{5}{9}$

A **Find the rational number that lies halfway between : $\frac{1}{2}$ and $\frac{4}{5}$**

Solution

The number $= \left(\frac{1}{2} + \frac{4}{5}\right) \div 2 = \frac{13}{20}$

B **Find a rational number lying at :**

(1) One fourth of the way between $\frac{5}{7}, -\frac{3}{7}$ *from the side of the smaller number.*

Solution

The distance between the two numbers

$= |\frac{5}{7} - (-\frac{3}{7})| = |\frac{5}{7} + \frac{3}{7}| = \frac{8}{7}$

Then the number $= -\frac{3}{7} + \frac{1}{4} \times \frac{8}{7} = -\frac{3}{7} + \frac{2}{7} = -\frac{1}{7}$

C One fifth of the way between $-\frac{1}{2}, -\frac{2}{5}$ *from the side of the greater number.*

Solution

The distance between the two numbers

$= |-\frac{1}{2} - (-\frac{2}{5})| = |-\frac{1}{2} + \frac{2}{5}| = |-\frac{5}{10} + \frac{4}{10}| = \frac{1}{10}$

Then the number

$= -\frac{4}{10} - \frac{1}{5} \times \frac{1}{10} = -\frac{4}{10} - \frac{1}{50} = \frac{-20-1}{50} = -\frac{21}{50}$

D One tenth of the way between $\frac{5}{6}, \frac{2}{3}$ *from the side of the smaller number.*

Solution

The distance between the two numbers

$= |\frac{5}{6} - \frac{2}{3}| = |\frac{5}{6} - \frac{4}{6}| = \frac{1}{6}$

Then the number $= \frac{4}{6} + \frac{1}{10} \times \frac{1}{6} = \frac{4}{6} + \frac{1}{60}$

$= \frac{40+1}{60} = \frac{41}{60}$

E Find the number one fourth of the way between $-\frac{1}{4}$ and $-\frac{7}{8}$ from the side of the smaller number.

F Find the number that lies one third of the way between $\frac{1}{4}$ and $\frac{7}{8}$ from the side of the smaller one.

G **Find the rational number that lies half way between : $\frac{1}{2}, \frac{1}{5}$**

H **Find the rational number that lies halfway between : $\frac{1}{2}$ and $\frac{4}{5}$**

Find three rational numbers lying between

A $\frac{1}{4}$ and $\frac{1}{5}$

First $\frac{1 \times 5}{4 \times 5} = \frac{5 \times 10}{20 \times 10} = \frac{50}{100}$

Second $\frac{1 \times 4}{1 \times 5} = \frac{4 \times 10}{20 \times 10} = \frac{40}{100}$

three rational numbers $= \frac{41}{100}$ / $\frac{42}{100}$ / $\frac{43}{100}$

B **Find three rational numbers between :** $\frac{1}{2}$ and $\frac{1}{3}$

First $\frac{1 \times 3}{2 \times 3} = \frac{3 \times 10}{6 \times 10} = \frac{30}{60}$

Second $\frac{1 \times 2}{3 \times 2} = \frac{2 \times 10}{6 \times 10} = \frac{20}{60}$

three rational numbers $= \frac{21}{60}$ / $\frac{22}{60}$ / $\frac{23}{60}$

C **Write three rational numbers between :** $\boxed{\frac{4}{9} \text{ and } \frac{5}{6}}$

First $\frac{4 \times 6}{9 \times 6} = \frac{24}{54}$

Second $\frac{5 \times 9}{6 \times 9} = \frac{45}{54}$

three rational numbers $\frac{24}{54}$ / $\frac{25}{54}$ / $\frac{26}{54}$

A

What is the increase of :

$3x^2-5x+2$ than $7x^2-x-3$?

Solution

$$\begin{array}{r} 3x^2-5x+2 \\ 7x^2-x-3 \\ \hline -4x^2-4x+5 \end{array}$$

Increase في حاله
يبقي كما هو و نغير
ال بعد than

What is the increase of :

$4x^2-6x+5$ **than** $7x^2-x-9$

Solution

B

Add the two expressions

$7x-3y-1$ **and** $2x+5y+3$

في حاله الجمع والطرح
متجيش جمب الاسس و الرموز ي حمار تنزل
ذي ما هي ما عدا الصفر

Solution

$$\begin{array}{r} 7x-3y-1 \\ 2x+5y+3 \\ \hline 9x+2y+2 \end{array}$$

C

Add : $2x-6z+y$, $3y+2z-5x$

Add : $3x^2-5x+1$ **and** x^2+x+3

Add : $5x^2+y^2-3xy$ **and** $x^2-2xy+3y^2$

Example 2 **Add the following expressions :**

$3x^3-4x^2+2x-1$, $5x^2-2x^3+3$ **and** $2-3x+x^2$

The first expression	:	$3x^3-4x^2+2x-1$
The second expression	:	$-2x^3+5x^2+3$
The third expression	:	$+x^2-3x+2$
The sum	=	x^3+2x^2-x+4

A **Example 4** **Subtract :** $5x - 3y + 2z$ **from** $2y - z + 7x$

Solution

$$
\begin{array}{rl}
: & 2y \quad -z \quad +7x \\
: & -3y + 2z + 5x \\
\hline
= & 5y - 3z + 2x
\end{array}
$$

في حالة subtract from
ال بعد from
يكتب في السطر الاول
واغير اشاره الاول

B **Subtract :** $y^3 + 5y^2 - 5y$ **from** $2y - y^3 + 5y^2$

Subtract : $5x^2 + y^2 - 3xy + 1$ **from** $6x^2 - 2xy + 3y^2$

Subtract : $-2x^2 - 5xy + 4y^2$ **from** $3x^2 + 2xy + 4y^2$

A	• $2a \times 5b = (2 \times 5) \times (a \times b) = 10ab$
B	• $(5x^2) \times (3x) = (5 \times 3) \times (x^2 \times x) = 15x^3$

C

$$2x(3x + 5y) = (2x \times 3x) + (2x \times 5y)$$
$$= 6x^2 + 10xy$$

$$\begin{array}{r} 3x + 5y \\ \times\ 2x \\ \hline \end{array}$$

The product $= 6x^2 + 10xy$

Example 2 **Find by inspection the product of each of the following :**

$(2a + 3)(5a + 1)$

$(2a+3)(5a+1) =$ The first × The first + Product of means + Product of extremes + The second × The second

$= (2a \times 5a) + (3 \times 5a + 2a \times 1) + 3 \times 1$

$= 10a^2 + (15a + 2a) + 3$

$= 10a^2 + 17a + 3$

Example 3 **Find the expansion of each of the following :**

1 $(3a + 5)^2$ **2** $(2x - 3y)^2$

Solution

1 $(3a+5)^2 = (3a)^2 + (2 \times 3a \times 5) + (5)^2$

$= 9a^2 + 30a + 25$

2 $(2x - 3y)^2 = (2x)^2 - (2 \times 2x \times 3y) + (3y)^2$

$= 4x^2 - 12xy + 9y^2$

Example 4 **Find the product of each of the following :**

1 $(2\ell - 5)(2\ell + 5)$

2 $(5x + 3y)(5x - 3y)$

3 $(a^2 + 2b)(a^2 - 2b)$

4 $\left(\frac{1}{3}a - \frac{2}{5}b\right)\left(\frac{1}{3}a + \frac{2}{5}b\right)$

Solution

1 $(2\ell - 5)(2\ell + 5) = (2\ell)^2 - (5)^2 = 4\ell^2 - 25$

2 $(5x + 3y)(5x - 3y) = (5x)^2 - (3y)^2 = 25x^2 - 9y^2$

3 $(a^2 + 2b)(a^2 - 2b) = (a^2)^2 - (2b)^2 = a^4 - 4b^2$

4 $\left(\frac{1}{3}a - \frac{2}{5}b\right)\left(\frac{1}{3}a + \frac{2}{5}b\right) = \left(\frac{1}{3}a\right)^2 - \left(\frac{2}{5}b\right)^2 = \frac{1}{9}a^2 - \frac{4}{25}b^2$

Simplify : $(y-5)(y+2)$

Solution $(y \otimes y)\ (2 \otimes y\ -5 \otimes y)\ (-5 \otimes 2)$ $\equiv y^2 - 3y - 10$

$y^2 \quad 2y - 5y = -3y \quad -10$

Simplify to the simplest form : $(2x-3)(2x+3)+7$

Solution $(2x \otimes 2x)(3 \otimes -3) = 4x^2 - 9 + 7 = 4x^2 - 2$

Simplify : $(x+2)^2 + (x-2)(x+2)$

Solution $(x \otimes x) + (2 \otimes x \otimes 2)\ (2 \otimes 2) = x^2 + 4x + \cancel{4}$

$(x \otimes x) \qquad (2 \otimes -2) = x^2 \qquad \cancel{-4}$

$2x^2 + 4x$

Find : $(2x-y)(2x+y)$ Solution $4x^2 - y^2$

Simplify : $(x+3)^2 - 9$ **, then find the numerical value when** $x = 3$

Solution **The expression** $= x^2 + 6x + 9 - 9 = x^2 + 6x$

الثاني مرتين | الاول ف الثاني ف 2 | الاول مرتين **The numerical value** $= 3^2 + 6 \times 3 = 9 + 18 = 27$

$(3b-4)(3b+4)+5$, then find the numerical value of the result when $b = -2$

Solution The expression $= 9b^2 - 16 + 5 = 9b^2 - 11$

The numerical value $= 9 \times (-2)^2 - 11 = 9 \times 4 - 11$

$= 36 - 11 = 25$

Simplify to the simplest form : $(x+3)^2 - (x+3)(x-3)$

$= x^2 + 6x + 9 - (x^2 - 9) = \cancel{x^2} + 6x + 9 \cancel{- x^2} + 9 = 6x + 18$

Simplify : $(2a - 3)(2a + 3) + 7$, then find the value of the result when $a = 1$

Solution The expression $= 4a^2 - 9 + 7 = 4a^2 - 2$

The numerical value $= 4 \times 1^2 - 2 = 4 - 2 = 2$

Simplify : $2a(a - 4b) + 4b(2a - 3b)$ **, then find the value of the result at :** $a = 2$, $b = -1$

Solution The expression $= 2a^2 - 8ab + 8ab - 12b^2$

$= 2a^2 - 12b^2$

The numerical value $= 2 \times 2^2 - 12 \times (-1)^2$

$= 2 \times 4 - 12 \times 1 = 8 - 12 = -4$

Simplify to the simplest form : $(x - 5)^2 + 10x$

Solution $= x^2 \cancel{- 10x} + 25 \cancel{+ 10x} = x^2 + 25$

Find the product of : $(3x - 4y)(2x + 5y)$

$$\begin{array}{l} \quad (3x - 4y) \\ \times \\ \quad (2x + 5y) \\ \hline \end{array}$$

(2x⊗ 3x)(2x⊗-4y)

(5x⊗ 3x) (5y ⊗ - 4y)

$6x^2 + 7xy - 20y^2$

Simplify to the simplest form : $(x - 3)(x + 3) + 9$, then calculate the numerical value of the result when $x = 5$

Simplify : $3\,a\,(a - b) + 4\,a\,(2\,a + b)$ in the simplest form.

Solution $3\,a^2 - 3\,a\,b + 8\,a^2 + 4\,a\,b = 11\,a^2 + a\,b$

Simplify : $(x - 3)\,(x + 3) - 9\,(x - 1)$

(2) $x^2 - 9 - 9\,x + 9 = x^2 - 9\,x$

Use the distributive property to find : $\frac{17}{12} \times \frac{23}{45} + \frac{7}{12} \times \frac{23}{45} - 2 \times \frac{23}{45}$

Solution $\left(\frac{17}{12} + \frac{7}{12} - 2\right) \times \frac{23}{45} = \left(\frac{24}{12} - 2\right) \times \frac{23}{45} = \text{zero}$

Simplify : $(2\,x + 5)^2 - 4\,x^2 - 10\,x$

Solution $4\,x^2 + 20\,x + 25 - 4\,x^2 - 10\,x = 10\,x + 25$

Simplify to the simplest form : $(x - 3)\,(x + 3) + 9$, then calculate the numerical value of the result when $x = 5$

Simplify to the simplest form : $(2\,x - 3)\,(2\,x + 3) + 7$, then calculate the numerical value of the result when : $x = -1$

Simplify to the simplest form : $(x + 2)^2 - (x + 2)\,(x - 2)$

Simplify the following expression to its simplest form :
$(x - 2)^2 - (x + 3)\,(x - 3) + 5\,(2\,x + 1)$

Find by inspection method the product of : $(x - 2)\,(x + 2)$

Find the product of : $(2\,x - 3\,y)\,(3\,x + 7\,y)$

Find the product of : $(3\,x - 4\,y)\,(2\,x + 5\,y)$

Simplify to the simplest form : $(x + 3)^2 - (x + 3)\,(x - 3)$

Simplify to the simplest form :
$3\,(1 - 2\,a) - (a^2 - 5\,a + 3) + 2\,a\,(a + 3)$, then find the numerical value when $a = 2$

Simplify to the simplest form : $(x - 5)^2 + 10\,x$

Simplify : $(y - 5)\,(y + 2)$

Example 1 **Find the quotient of dividing :**

$5a - 10a^2 + 6a^3 + 3$ **by** $3 + 2a^2 - 4a$ where the divisor $\neq 0$

Solution

$$\begin{array}{l|l}
2a^2 - 4a + 3 & 6a^3 - 10a^2 + 5a + 3 \\
\hline
3a + 1 & \ominus\ \oplus\ \ominus \\
 & 6a^3 - 12a^2 + 9a \\
\cline{2-2}
 & \quad 2a^2 - 4a + 3 \\
 & \quad \ominus\ \oplus\ \ominus \\
 & \quad 2a^2 - 4a + 3 \\
\cline{2-2}
 & \quad 00 \quad 00 \quad 00
\end{array}$$

Notice that :
Each of the dividend and the divisor is in a descending order according to the power of " a".

i.e. The quotient = $3a + 1$

Example 2 **Find the quotient of dividing :**

$x^3 + x + 10$ **by** $x + 2$ **where** $x \neq -2$

Solution

$$\begin{array}{l|l}
x + 2 & x^3 + \qquad\quad x + 10 \\
\hline
x^2 - 2x + 5 & \ominus\ \ominus \\
 & x^3 + 2x^2 \\
\cline{2-2}
 & -2x^2 + x + 10 \\
 & \oplus\ \oplus \\
 & -2x^2 - 4x \\
\cline{2-2}
 & \quad 5x + 10 \\
 & \quad \ominus\ \ominus \\
 & \quad 5x + 10 \\
\cline{2-2}
 & \quad 00 \quad 00
\end{array}$$

Notice that :
There is no term with x^2 in dividend , so we leave its place empty.

i.e. The quotient = $x^2 - 2x + 5$

Example 3 **If $(x - 1)$ is one of the factors of $(x^2 + 5x - 6)$, then find the other factor.**

Solution

The other factor is the quotient of dividing

$x^2 + 5x - 6$ by $(x - 1)$

i.e. The other factor is $(x + 6)$

$$\begin{array}{l|l}
x - 1 & x^2 + 5x - 6 \\
\hline
x + 6 & \ominus\ \oplus \\
 & x^2 - x \\
\cline{2-2}
 & \quad 6x - 6 \\
 & \quad \ominus\ \oplus \\
 & \quad 6x - 6 \\
\cline{2-2}
 & \quad 00 \quad 00
\end{array}$$

Divide : $(x^2 + 5x + 6)$ **by** $(x + 2)$

$$\begin{array}{r|l} & x + 3 \\ \hline x+2 & x^2 + 5x + 6 \\ & \ominus x^2 \ominus 2x \\ \hline & 3x + 6 \\ & \ominus 3x \ominus 6 \\ \hline & 0 \quad 0 \end{array}$$

The quotient $= x + 3$

$x^2 + 5x + 6$ **by** $x + 3$

$$\begin{array}{r|l} & x + 2 \\ \hline x+3 & x^2 + 5x + 6 \\ & \ominus x^2 \ominus 3x \\ \hline & 2x + 6 \\ & \ominus 2x \ominus 6 \\ \hline & 0 \quad 0 \end{array}$$

The quotient $= x + 2$

Divide : $6x^2 + 13xy + 6y^2$ **by** $2x + 3y$

$$\begin{array}{r|l} 2x + 3y & 6x^2 + 13xy + 6y^2 \\ \hline 3x + 2y & \ominus 6x^2 \ominus 9xy \\ \hline & 4xy + 6y^2 \\ & \ominus 4xy \ominus 6y^2 \\ \hline & 0 \quad 0 \end{array}$$

The quotient $= 3x + 2y$

$16x^2 - 24xy + 9y^2$ **by** $4x - 3y$

$$\begin{array}{r|l} 4x - 3y & 16x^2 - 24xy + 9y^2 \\ \hline 4x - 3y & \ominus 16x^2 \oplus 12xy \\ \hline & -12xy + 9y^2 \\ & \oplus 12xy \ominus 9y^2 \\ \hline & 0 \quad 0 \end{array}$$

The quotient $= 4x - 3y$

Divide : $(x^2 - 5x + 6)$ **by** $(x - 3)$ (where $x \neq 3$)

Find the quotient of : $x^2 - 2x - 8$ **by** $(x - 4)$ (where $x \neq 4$)

Find the quotient of : $x^3 + 3x^2 - x - 3$ **by** $x^2 - 1$ (where $x^2 - 1 \neq 0$)

Divide : $6x^2 + 13xy + 6y^2$ **by** $2x + 3y$ (where $2x + 3y \neq 0$)

Divide : $6x^2y - 9xy^2 + 24xy$ by xy

Solution $\frac{6x^2y}{xy} - \frac{9xy^2}{xy} + \frac{24xy}{xy}$ = 6x +9y 24

Divide : $30x^3 - 25x^2 + 15x$ by $5x$ (where $x \neq 0$)

Solution $\frac{30x^3}{5x} - \frac{25x^2}{5x} + \frac{15x}{5x} = 6x^2 - 5x + 3$

Find the quotient of : $30a^2b^3 - 25a^3b^2 + 35ab$ by $5ab$

Solution $\frac{30a^2b^3}{5ab} - \frac{25a^3b^2}{5ab} + \frac{35ab}{5ab} = 6ab^2 - 5a^2b + 7$

Divide : $x^3y^3 - 4x^2y^2 + 6xy^2$ by xy (where : $xy \neq 0$)

Solution $\frac{x^3y^3}{xy} - \frac{4x^2y^2}{xy} + \frac{6xy^2}{xy} = x^2y^2 - 4xy + 6y$

The necessary condition to make $\frac{5}{x-3}$ a rational number is ....

(a) $x = -3$ (b) $x = 3$ (c) $x \neq 3$ (d) $x = 5$

$(a^2 + a) \div a =$ .... (where $a \neq 0$)

(a) a (b) 0 (c) 2 a + 1 (d) a + 1

$\frac{3x}{7} - \frac{x}{7} =$ ....

(a) $\frac{2}{7}$ (b) $\frac{x}{7}$ (c) $\frac{2x}{7}$ (d) $2x$

Divide : $2x^3 + 11x^2 + 12x - 9$ **by** $x + 3$

Solution

$$
\begin{array}{r|l}
 & 2x^2 + 5x - 3 \\
\hline
x+3 & 2x^3 + 11x^2 + 12x - 9 \\
 & \ominus\ 2x^3 \ominus 6x^2 \\
\hline
 & 5x^2 + 12x - 9 \\
 & \ominus\ 5x^2 \ominus 15x \\
\hline
 & -3x - 9 \\
 & \oplus\ -3x \oplus -9 \\
\hline
 & 0 \quad 0
\end{array}
$$

$$\frac{x+3}{2x^2+5x-3}$$

The quotient $= 2x^2 + 5x - 3$

Divide : $10x^4 - 5x^3$ **by** $5x^2$ (if $x \neq 0$)

Divide : $x^3y^3 - 4x^2y^2 + 6xy^2$ **by** xy (where : $xy \neq 0$)

(1) **Add :** $3x - 2y + 5$ **and** $2x + y - 3$

(2) **Divide :** $6x^3y^3 + 4xy^2$ **by** $2xy$ (where $xy \neq 0$)

Divide : $6x^3y^2 + 9x^2y^3$ **by** $3x^2y^2$ (where $x \neq 0$, $y \neq 0$)

Divide : $30x^3 - 25x^2 + 15x$ **by** $5x$ (where $x \neq 0$)

Find the quotient of : $(x^2 + 5x + 6)$ **by** $(x + 2)$ (where $x \neq -2$)

Find the quotient of : $(x^3 - 6x^2 + 11x - 6)$ **by** $(x - 3)$ (where $x \neq 3$)

Divide : $2x^3 + 11x^2 + 12x - 9$ **by** $x + 3$ (where $x \neq -3$)

Find the quotient of : $13x + 15 + 2x^2$ **by** $x + 5$ (where $x \neq -5$)

Find the value of k which makes the expression : $2x^3 - x^2 - 5x + k$ **divided by** $2x - 3$

Find the quotient : $6x^2 - xy - 15y^2$ **by** $2x + 3y$ (where $2x + 3y \neq 0$)

Factorize by using (H.C.F) : $3a(a-2b)+7b(a-2b)$

Solution $(a-2b)(3a+7b)$

Factorize by taking the H.C.F : $15xy^3+20x^2y-25xy$

Solution $5xy(3y^2+4x-5)$

Factorize the expression by identifying the H.C.F : $12y^3+18y^2$

Solution $6y^2(2y+3)$

If $x+4=4$ **, then find :** $x(x+4)+4(4+x)$

Where $x+4=4$, then $x=0$

, then the value $=0\times(0+4)+4\times(4+0)$

$=0\times 4+4\times 4=0+16=16$

$(x+4)(x+4)=4\times 4=16$

Factorize by identifying the H.C.F : $3x^2+15xy$

Solution $3x(x+5y)$

Subtract : $-5x$ **from** $3x$ Solution $3x+5x=8x$

Factorize by identifying the H.C.F. : $12x^3+8x^2-4x$

Solution $4x(3x^2+2x-1)$

By using the highest common factor , find the result of : $(17)^2-8\times 17+17$

Solution $17(17-8+1)=17\times 10=170$

$\frac{6}{37}\times 7+\frac{6}{37}\times 5+\frac{6}{37}\times(-11)$ Solution $\frac{6}{37}\left(7+5+(-11)\right)=\frac{6}{37}\times 1=\frac{6}{37}$

Factorize by identifying the H.C.F : $3a(a-2b)-6b(a-2b)$

, then find the numerical value of the result when $a-2b=\left|\frac{-1}{3}\right|$

Solution The expression $=3(a-2b)(a-2b)=3(a-2b)^2$

The numerical value $=3\times(\frac{1}{3})^2=3\times\frac{1}{9}=\frac{1}{3}$

Factorize by identifying the H.C.F : $6x^4y^3-12x^3y^4+2x^3y^3$

Solution $2x^3y^3(3x-6y+1)$

Factorize by taking out the H.C.F : $27x^3-18x^2+6x$

Solution $3x(9x^2-6x+2)$

Simplify : $3(1-2x)-(x^2-5x+3)+2x(x+3)$

, then find the numerical value of the result when $x=-1$

Solution $3-6x-x^2+5x-3+2x^2+6x=x^2+5x$

at $x=-1$

The numerical value $=(-1)^2+5(-1)=1-5=-4$

Factorize by identifying the H.C.F : $a(a-2b)-2b(a-2b)$

Solution $(a-2b)(a-2b)=(a-2b)^2$

Factorize by identifying the H.C.F : $a(a-2b)-2b(a-2b)$

, then find the numerical value of the result when $(a-2b)=\frac{1}{3}$

Solution $(a-2b)(a-2b)=(a-2b)^2$

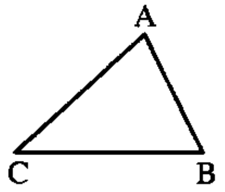
at $(a-2b)=\frac{1}{3}$ The numerical value $=\left(\frac{1}{3}\right)^2=\frac{1}{9}$

Sheet (5)
Congruent triangles

We know that any triangle has three sides and three angles which are known as the six elements of the triangle.

For example :

Δ ABC has three sides which are : $\overline{AB}$, $\overline{BC}$ and $\overline{AC}$ and
it has three angles which are : $\angle A$, $\angle B$ and $\angle C$



Therefore :

The two triangles are congruent if each element of the 6 elements of one of them is congruent to the corresponding element in the other triangle and vice versa.

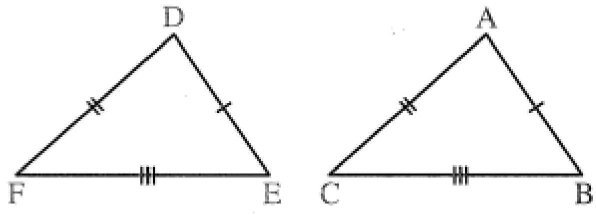
- To test whether two triangles are congruent or not, you don't need to test all the three sides and the three angles.

The cases of congruence of two triangles

Case (1)	Case (2)	Case (3)	Case (4)
Two sides and the included angle	Two angles and one side	Three sides	Hypotenuse and one side in the right-angled triangle
S. A. S.	A. S. A.	S. S. S.	R. H. S.
Two triangles are congruent if two sides and the included angle of one triangle are congruent to the corresponding parts of the other triangle	Two triangles are congruent if two angles and the side drawn between their vertices of one triangle are congruent to the corresponding parts of the other triangle	Two triangles are congruent if each side of one triangle is congruent to the corresponding side of the other triangle	Two right-angled triangles are congruent if the hypotenuse and a side of one triangle are congruent to the corresponding parts of the other triangle

Remark

If each angle of one triangle is congruent to the corresponding angle of the other triangle , it is not necessary for the two triangles to be congruent.



Prove that $\Delta\ ABC \equiv \Delta\ DEF$

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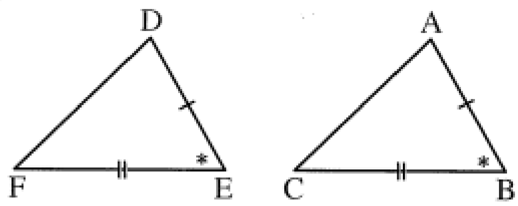

Prove that $\Delta\ ABC \equiv \Delta\ DEF$

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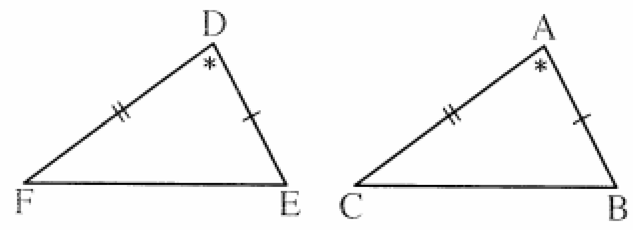
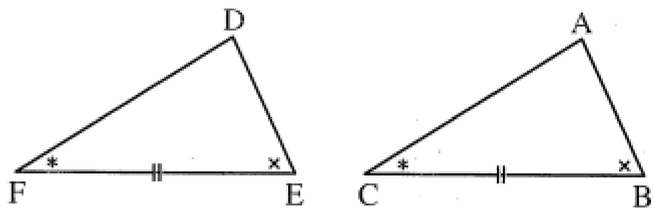
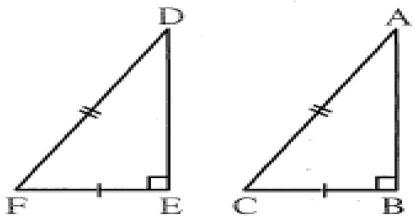


Prove that $\Delta\ ABC \equiv \Delta\ DEF$



Prove that $\Delta\ ABC \equiv \Delta\ DEF$



Prove that $\Delta ABC \equiv \Delta DEF$

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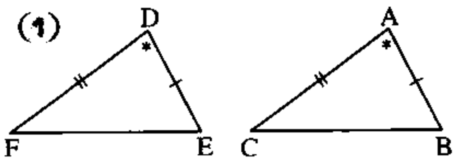
[1] In each of the following figures, show if the two triangles are congruent or not. If they are congruent, name the case of congruence. If they aren't congruent, give reason. (given that the similar signs denoted the congruency of the elements marked by these signs).

(1)



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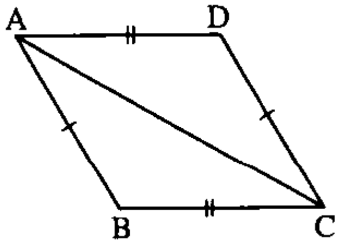

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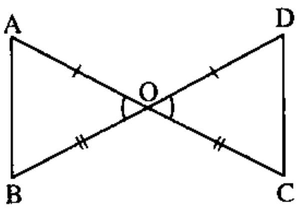

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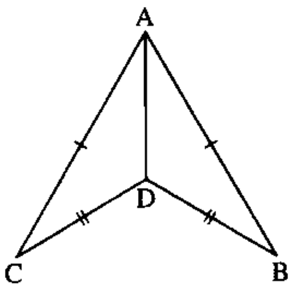

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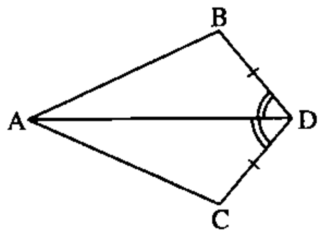

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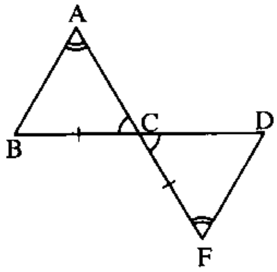

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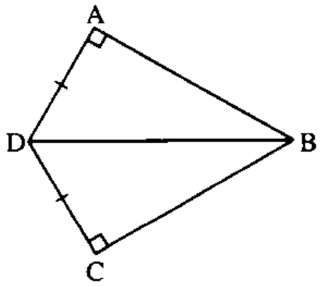

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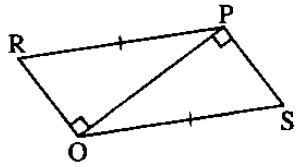

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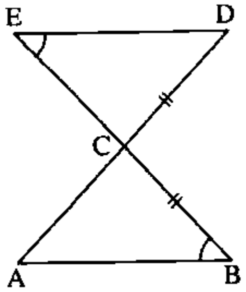

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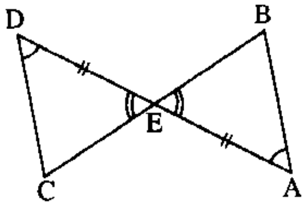

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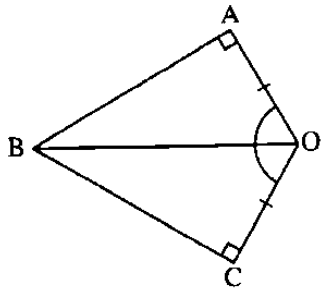

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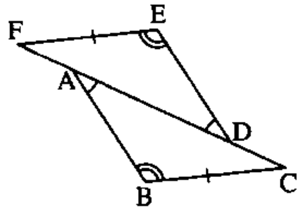

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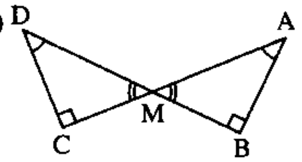

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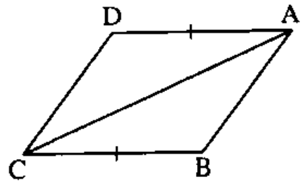

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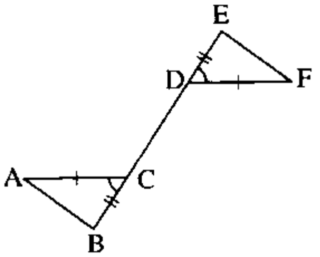

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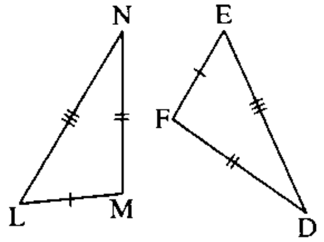

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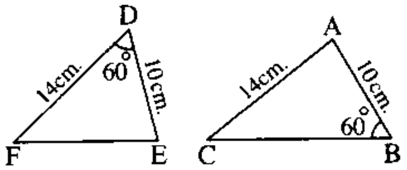

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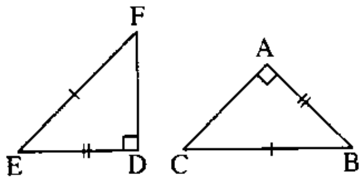

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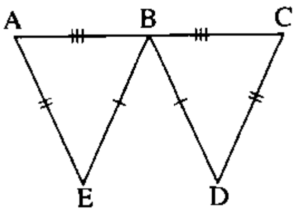

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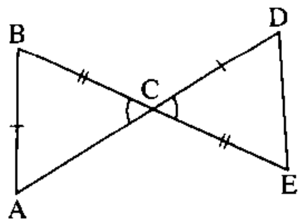

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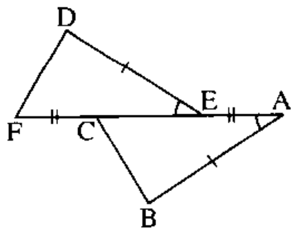

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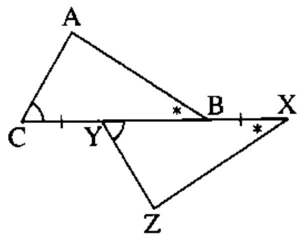

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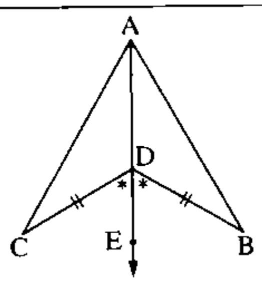

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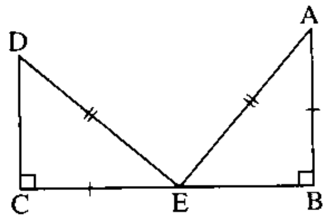

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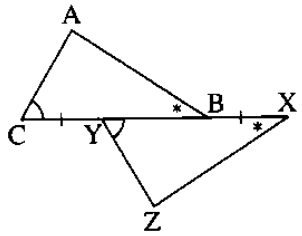

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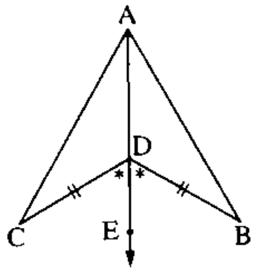

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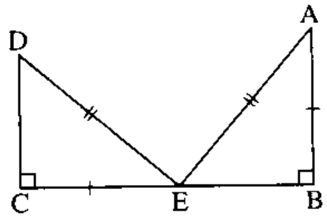

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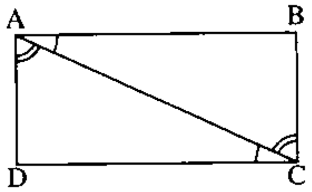

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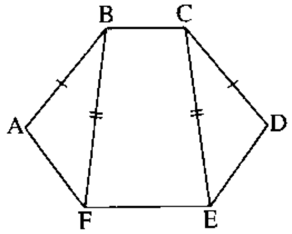
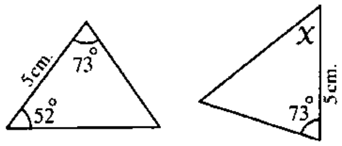


[2] Answer the following:

(1) **In the opposite figure:**

These triangles are congruent

, then $X =$ ....°



(2) **In the opposite figure:**

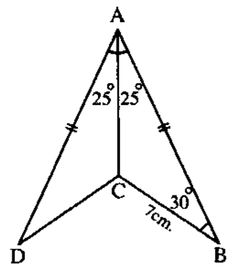
If : AB = AD , BC = 7 cm. , m (∠ BAC) = m (∠ DAC) = 25°

and m (∠ B) = 30°

Complete the following :

(1) Δ ACB ≡ Δ ....   (2) m (∠ D) = ...°

(3) CD = ... cm.   (4) m (∠ ACD) = ...°

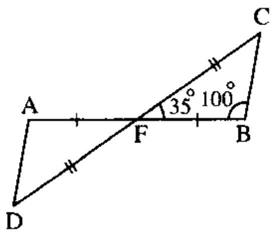


(3) **In the opposite figure:**

If : $\overline{CD} \cap \overline{BA} = \{F\}$, FA = FB , CF = FD ,

m (∠ CFB) = 35° and m (∠ B) = 100° ,

then m (∠ D) = ...°



(4) **In the opposite figure:**

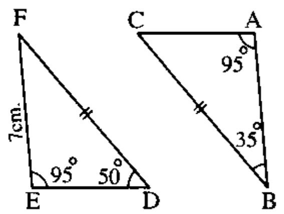
If : BC = FD , m (∠ A) = m (∠ E) = 95° ,

m (∠ B) = 35° , m (∠ D) = 50° and FE = 7 cm.

Complete the following :

(1) m (∠ C) = ...°   (2) m (∠ F) = ...°   (3) Δ ABC ≡ ....

(4) $\overline{AC}$ ≡ ....   (5) AB = ... cm.



(5) **In the opposite figure:**

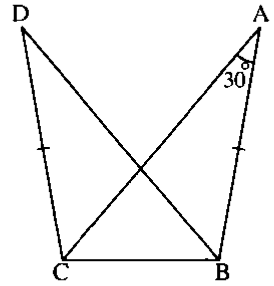
If : AB = DC , AC = DB and m (∠ A) = 30°

Complete the following :

(1) Δ ABC ≡ Δ ………

(2) m (∠ D) = ………°

(3) m (∠ DBC) = m (∠ ………)



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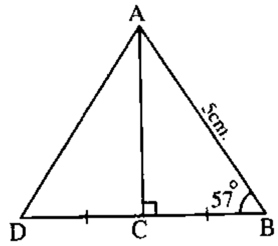
(6) **In the opposite figure:**

C is the midpoint of $\overline{BD}$, $\overline{AC} \perp \overline{BD}$,

AB = 5 cm. and m (∠ B) = 57°

Find : (1) The length of $\overline{AD}$

(2) m (∠ DAC)



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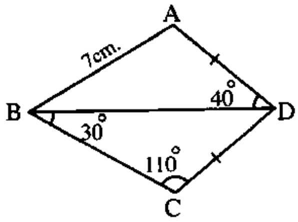
(7) **In the opposite figure:**

AD = DC , m (∠ ADB) = 40° , m (∠ DBC) = 30° ,

m (∠ BCD) = 110° and AB = 7 cm.

Find : (1) The length of $\overline{BC}$ (2) m (∠ BAD)



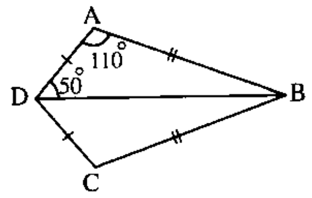
(8) In the opposite figure:

$BA = BC$, $DA = DC$,

$m(\angle ADB) = 50°$ and

$m(\angle BAD) = 110°$

Find : $m(\angle ABC)$



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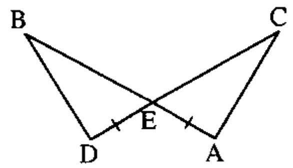
(9) In the opposite figure:

$\overline{AB} \cap \overline{CD} = \{E\}$, $AE = ED$ and $\angle A \equiv \angle D$

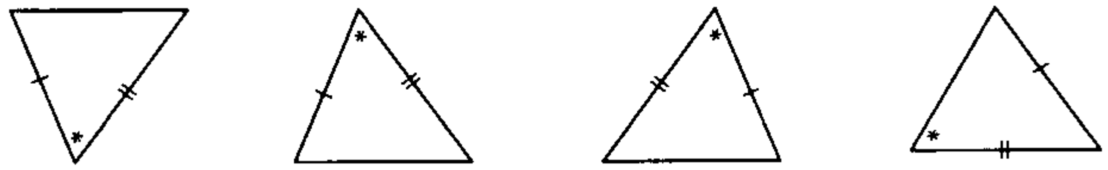
Is $\Delta ACE \equiv \Delta DBE$? Why ?

Prove that : $CE = EB$



[3] Choose the correct answer:

(1) The following triangles are congruent except ....

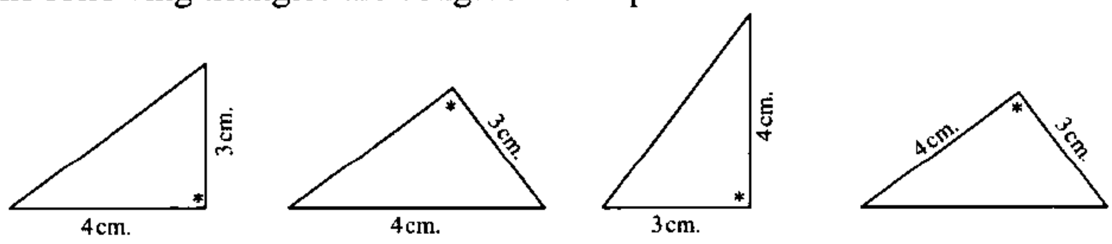


(a) (b) (c) (d)

(2) The following triangles are congruent except ....

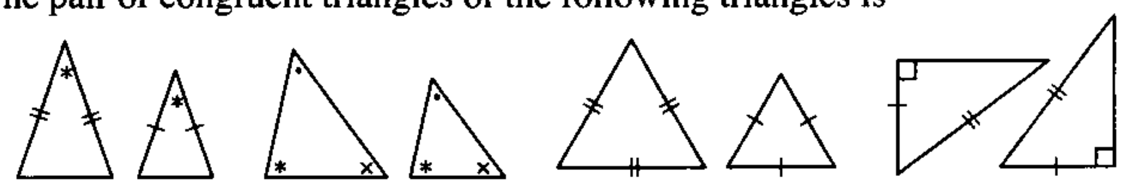
(a) (b) (c) (d)

(3) The following triangles are congruent except ....



(a) (b) (c) (d)

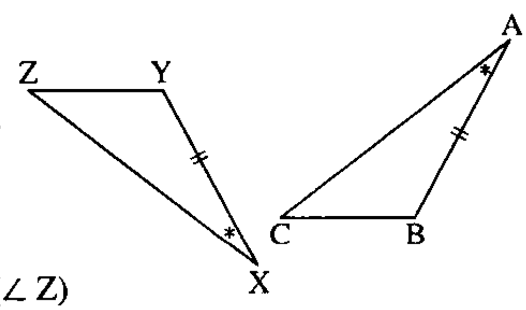
(4) The pair of congruent triangles of the following triangles is ....



(a) (b) (c) (d)

(5) **In the opposite figure :**

The necessary and enough condition which makes the two triangles ABC and XYZ be congruent is ....



(a) BC = YZ (b) AC = XZ

(c) m (∠ C) = m (∠ Z) (d) m (∠ B) = m (∠ Z)

[4] Complete the following:

(1) If : $\Delta ABC \equiv \Delta XYZ$, $m(\angle A) = 50^\circ$ and $m(\angle B) = 60^\circ$, then : $m(\angle Z) =$ ....°

(2) If : $\Delta ABC \equiv \Delta LMN$, $m(\angle L) = 40^\circ$ and $m(\angle B) = 90^\circ$, then : $m(\angle C) =$ ....°

(3) If : $\Delta ABC \equiv \Delta XYZ$ and $m(\angle A) + m(\angle B) = 120^\circ$, then : $m(\angle Z) =$ ....°

(4) If : $\Delta ABC \equiv \Delta DEF$ and $m(\angle C) = 90^\circ$, then : $m(\angle D) + m(\angle E) =$ ....°

(5) If : $\Delta ABC \equiv \Delta XYZ$, the perimeter of $\Delta ABC = 12$ cm. , $XY = 4$ cm. and $YZ = 5$ cm. , then : $AC =$ ....

(6) Any two triangles are congruent if each .... is congruent to its corresponding side in the other triangle.

(7) Any two triangles are congruent if two angles and .... in one of the triangles are congruent to their corresponding elements in the other.

(8) The diagonal of the rectangle divides its surface into two .... triangles.

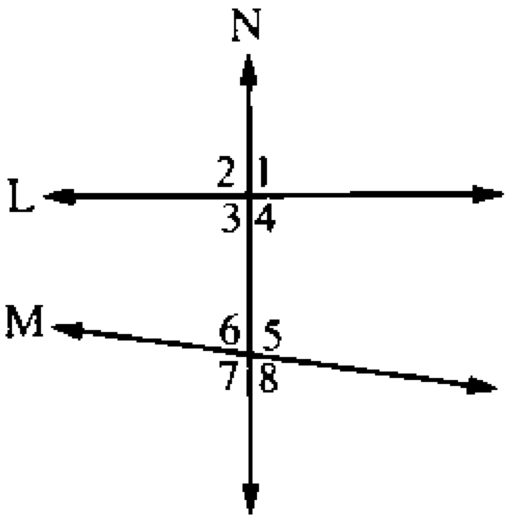
(9) If $\Delta ABC \equiv \Delta XYZ$, then $AB =$ .... and $m(\angle Z) = m(\angle$ ....)

(10) If : $AB = LM$, $BC = MN$ and $m(\angle B) = m(\angle M)$, then the two triangles .... and .... will be congruent.

Sheet [6]
Parallelism

Angles Formed from two straight lines and a transversal:

If a straight line N cuts two straight lines L and M as shown in the opposite figure, then we get eight angles.

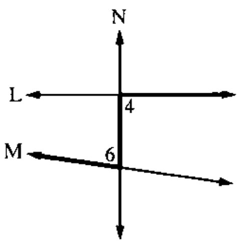


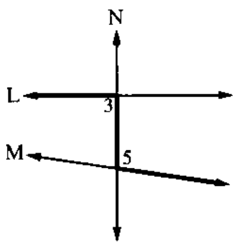
We can classify these angles into pairs of angles:

- Alternate angles.
- Corresponding angles.
- Interior angles on the same side of the transversal.

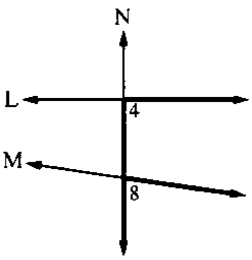
As follows

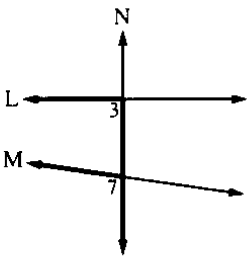
(1) Pairs of alternate angles:

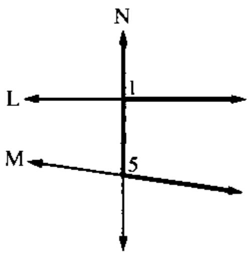


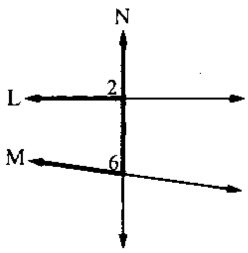


(2) Pairs of corresponding angles:

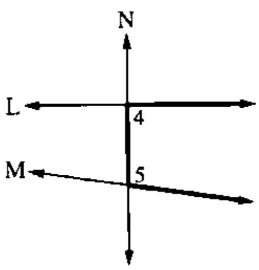


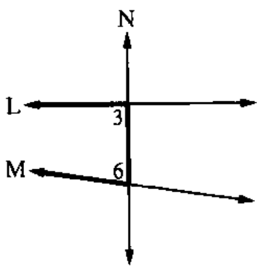






(3) Pairs of interior angles on the same side of the transversal





Relation between pairs of angles formed from two parallel straight lines and a transversal to them

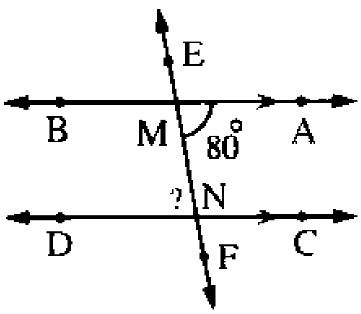
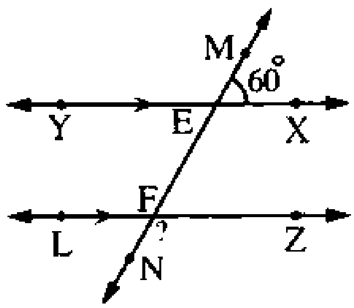
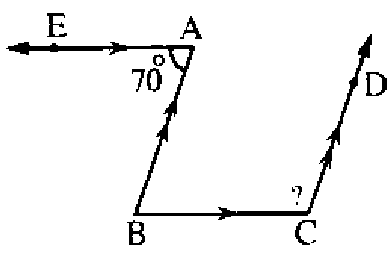
If a straight line intersects two parallel lines, then:

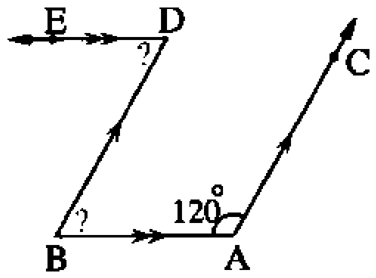
(1) Each two alternate angles are equal in measure.

(2) Each two corresponding angles are equal in measure.

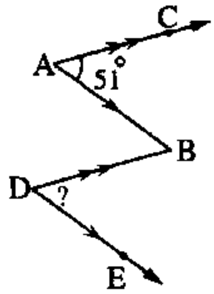
(3) Each two interior angles in the same side of the transversal are supplementary.

In each of the following figures, find the measure of the angle which is marked by (?) giving reason:

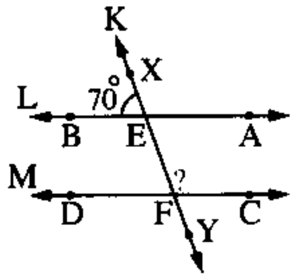
 (1)	 (2)	 (3)
....	....	....



(4)



(5)



(6)

The condition of parallelism of two straight lines

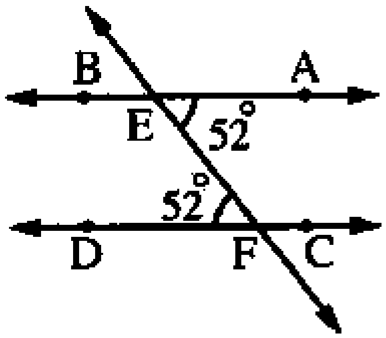
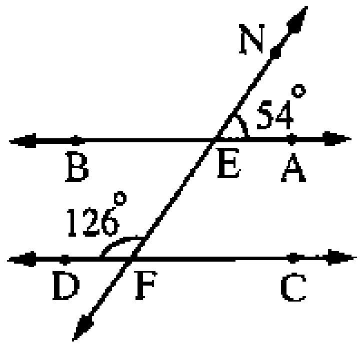
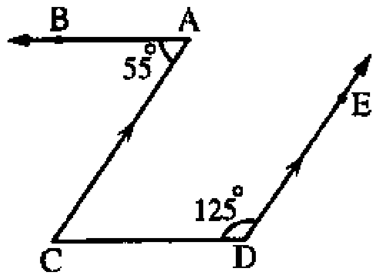
The two straight lines are parallel if a third straight line intersects them (as a transversal) and one of the following cases satisfied:

(1) Two alternate angles have the same measure.

(2) Two corresponding angles have the same measure.

(3) Two interior angles in the same side of the transversal are supplementary.

In each of the following figures, why is $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$?

 (1)	 (2)	 (3)

Geometric facts

(1) The perpendicular to one of two parallel straight lines is perpendicular to the other.

(2) If two straight lines are perpendicular to a third one, then the two straight lines are parallel.

(3) If two straight lines are parallel to a third one, then the two straight lines are parallel.

(4) If parallel straight lines divide a straight line into segments of equal lengths, then they divide any other line into segments of equal lengths.

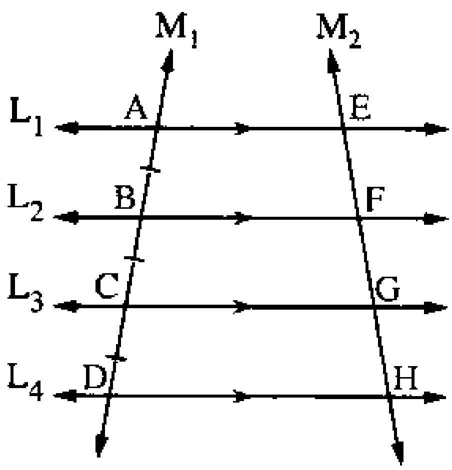
If L_1 // L_2 // L_3 // L_4,

and M_1 and M_2 are two transversal in which:

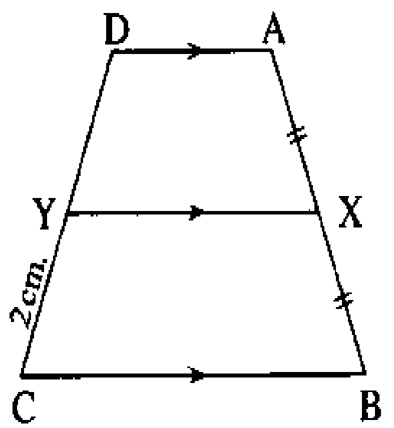
$AB = BC = CD$,

then:

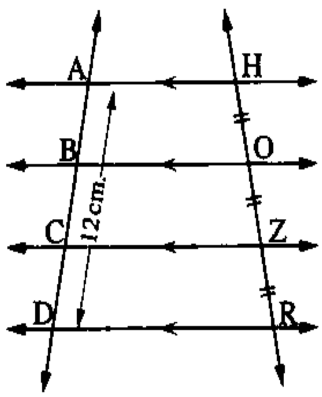
$EF = FG = GH$



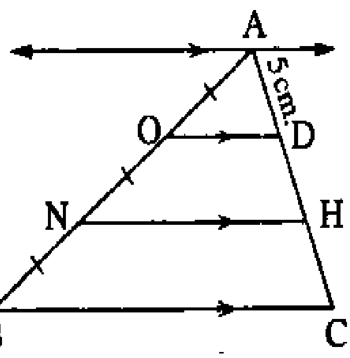
Complete using the given shown in the following figures:



DY = …………… cm



AC = ………… cm



AC = ………… cm

[1] Choose the correct answer:

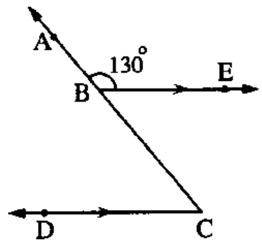
(1) **In the opposite figure:**

$B \in \overline{AC}$, $\overrightarrow{BE} \parallel \overrightarrow{CD}$ and m ($\angle$ ABE) = 130°

Then m ($\angle$ C) = ...

(a) 130°　　(b) 40°

(c) 50°　　(d) 90°



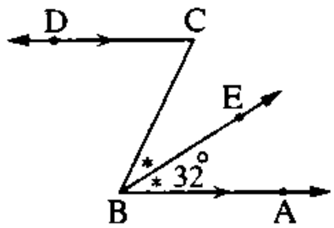
(2) **In the opposite figure:**

$\overrightarrow{BE}$ bisects $\angle$ ABC , $\overrightarrow{BA} \parallel \overrightarrow{CD}$ and

m ($\angle$ ABE) = 32° , then m ($\angle$ C) = ...

(a) 32°　　(b) 64°

(c) 60°　　(d) 80°



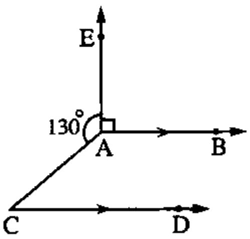
(3) **In the opposite figure:**

$\overrightarrow{AB} \parallel \overrightarrow{CD}$, m ($\angle$ EAC) = 130°

and m ($\angle$ EAB) = 90° , then m ($\angle$ C) = ...

(a) 90°　　(b) 130°

(c) 140°　　(d) 40°



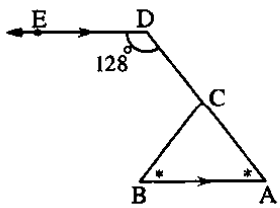
(4) **In the opposite figure:**

$\overrightarrow{AB} \parallel \overrightarrow{DE}$, m ($\angle$ D) = 128° ,

m ($\angle$ A) = m ($\angle$ B) and $C \in \overline{AD}$, then m ($\angle$ B) = ...

(a) 64°　　(b) 128°

(c) 52°　　(d) 26°



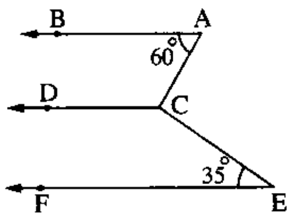
(5) **In the opposite figure:**

$\overrightarrow{AB} \parallel \overrightarrow{CD}$, $\overrightarrow{AB} \parallel \overrightarrow{EF}$, m ($\angle$ A) = 60° and

m ($\angle$ E) = 35° , then m ($\angle$ ACE) = ...

(a) 60°　　(b) 35°

(c) 95°　　(d) 85°



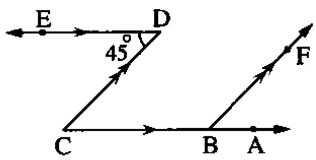
(6) **In the opposite figure:**

m ($\angle$ D) = 45° , $\overrightarrow{DE} \parallel \overrightarrow{CA}$ and

$\overline{CD} \parallel \overrightarrow{BF}$, then m ($\angle$ ABF) = ...

(a) 45°　　(b) 90°

(c) 135°　　(d) 40°



(7) In the opposite figure:

$\overline{AX} // \overline{BY} // \overline{CE} // \overline{DF}$,

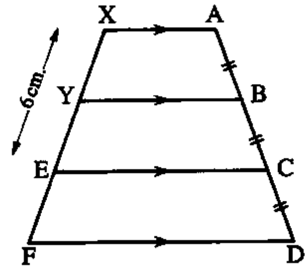
AB = BC = CD

and XE = 6 cm.

, then the length of $\overline{YF}$ = ...

(a) 3 cm. (b) 6 cm.

(c) 12 cm. (d) 9 cm.



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(8) In the opposite figure:

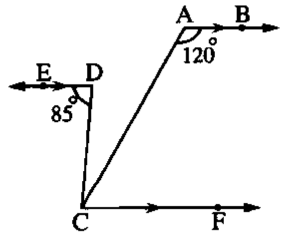
$\overrightarrow{AB} // \overrightarrow{CF} // \overrightarrow{DE}$,

m (∠ A) = 120° and m (∠ D) = 85° ,

then m (∠ ACD) = ...

(a) 60° (b) 85°

(c) 25° (d) 120°



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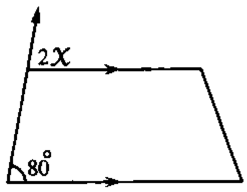
(9) In the opposite figure:

What is the value of X ?

(a) 40° (b) 60°

(c) 80° (d) 100°



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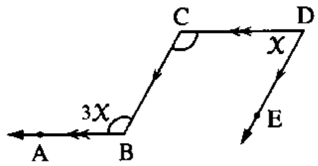
(10) In the opposite figure:

$\overline{CD} // \overrightarrow{BA}$, $\overrightarrow{DE} // \overline{CB}$

, then : X = ...

(a) 60° (b) 45°

(c) 120° (d) 90°



[2] Complete:

(1) 📖 The straight line which is perpendicular to one of two parallel straight lines is .... to the other straight line in the plane.

---

(2) 📖 If two straight lines are parallel to a third straight line , then they are ....

---

(3) If a straight line cuts two parallel straight lines , then each two alternate angles are ....

---

(4) If a straight line cuts two parallel straight lines , then each two corresponding angles are ....

(5) If a straight line cuts two parallel straight lines , then each two interior angles in the same side of the transversal are ....

(6) If a straight line cuts two straight lines and there are two corresponding angles having the same measure , then the two straight lines are ....

(7) If a straight line cuts two straight lines and there are two alternate angles having the same measure , then the two straight lines are ....

(8) If a straight line cuts two straight lines and there are two interior angles in the same side of the transversal are supplementary , then the two straight lines are ....

(9) If a straight line cuts several parallel lines and the intercepted parts of this transversal between these parallel straight lines are equal in length , then the intercepted parts for any transversal are ....

[3] Answer the following:

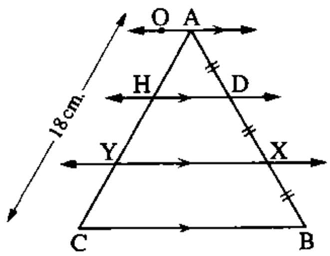
(1) In the opposite figure:

$\overleftrightarrow{AO} \parallel \overleftrightarrow{HD} \parallel \overleftrightarrow{YX} \parallel \overleftrightarrow{CB}$

, $AD = DX = XB$

and $AC = 18$ cm.

Find the length of $\overline{AY}$

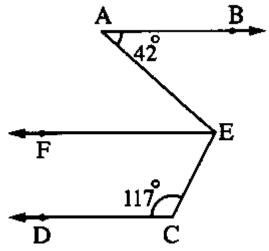


(2) In the opposite figure:

$\overrightarrow{AB} \parallel \overrightarrow{CD}$, $\overrightarrow{EF} \parallel \overrightarrow{CD}$

, m ($\angle$ A) = 42° and m ($\angle$ C) = 117°

Determine : m ($\angle$ AEC)

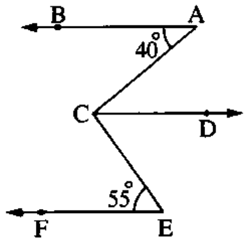


(3) In the opposite figure:

m ($\angle$ A) = 40° , m ($\angle$ E) = 55°

$\overrightarrow{AB} \parallel \overrightarrow{EF}$ and $\overrightarrow{AB} \parallel \overrightarrow{CD}$

Find : m ($\angle$ ACE)

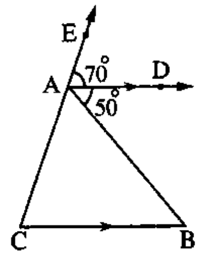


(4) In the opposite figure:

$\overrightarrow{AD} \parallel \overline{BC}$, $E \in \overrightarrow{CA}$,

m ($\angle$ DAE) = 70° and m ($\angle$ DAB) = 50°

Find the measures of the triangle ABC

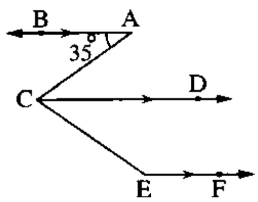


(5) In the opposite figure:

$\overleftrightarrow{AB} \parallel \overrightarrow{CD} \parallel \overrightarrow{EF}$, m ($\angle$ A) = 35° and

$\overrightarrow{CD}$ bisects $\angle$ ACE

Find : (1) m ($\angle$ DCE) (2) m ($\angle$ CEF)

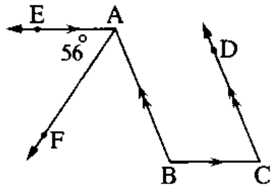


(6) In the opposite figure:

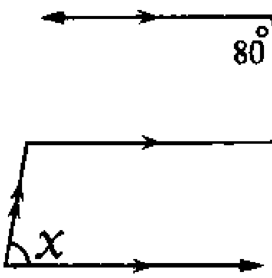
$\overrightarrow{AE} \parallel \overline{CB}$, $\overline{BA} \parallel \overrightarrow{CD}$,

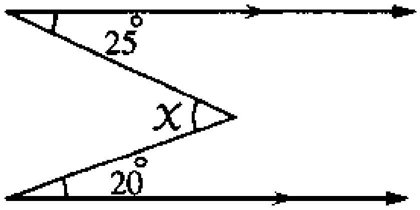
$\overrightarrow{AF}$ bisects $\angle$ BAE and m ($\angle$ EAF) = 56°

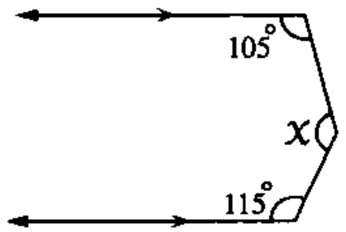
Find : m ($\angle$ C)

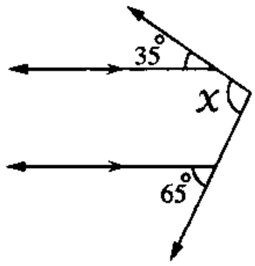


[4] Find the value of x:






105°
x
115°


35°
x
65°

4-4 Congruent triangles

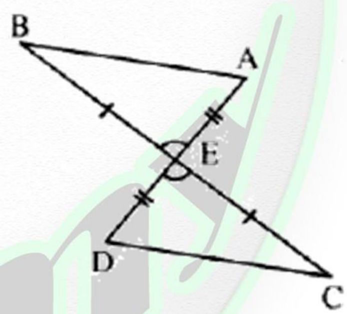
The cases of congruence of two triangles

- Two sides and the included angle
- Two angles and one side
- Three sides
- Hypotenuse and one side in the right-angled triangle

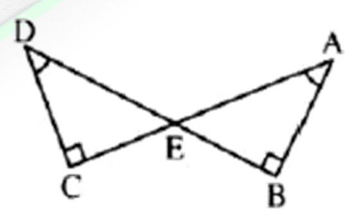
Example:

In each of the following figures , show if the two triangles are congruent or not. If they are congruent , name the case of congruence. If they are not congruent , give reason (Given that the similar signs denote the congruency of the elements marked by these signs).

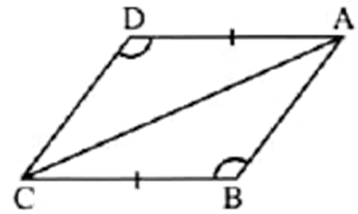
1) ....



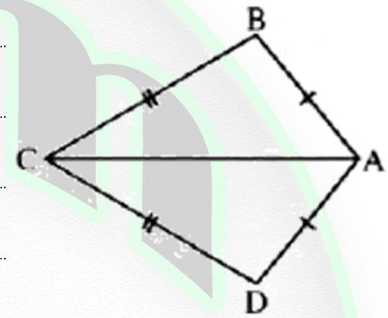
2) ....



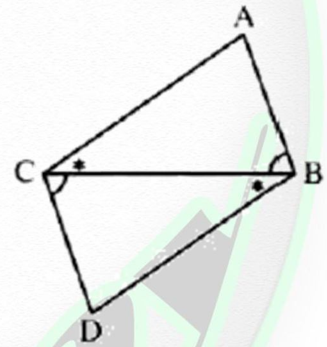
3)



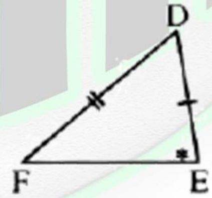
4)

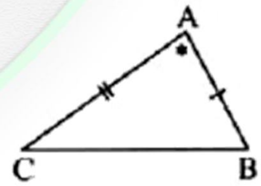


5)



6)





Example:

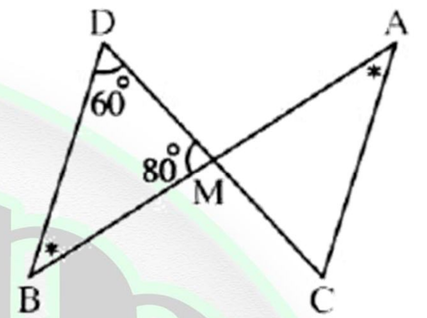
7) **In the opposite figure :**

$\overline{AB} \cap \overline{CD} = \{M\}$, M is the midpoint of $\overline{AB}$,

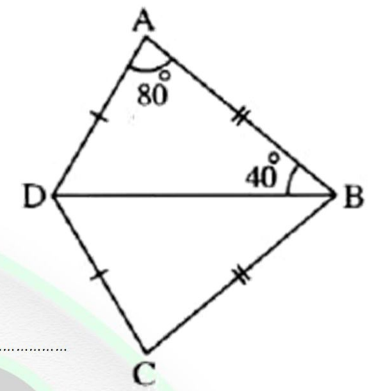
m (∠ A) = m (∠ B) , m (∠ D) = 60°

and m (∠ DMB) = 80°

Find : m (∠ C) with showing the steps of the solutions.



8) **In the opposite figure :**



BA = BC , DA = DC ,

m (∠ ABD) = 40° and m (∠ BAD) = 80°

Find : m (∠ ADC) with showing the steps of the solution.

....

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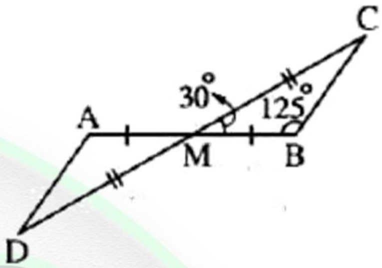
....

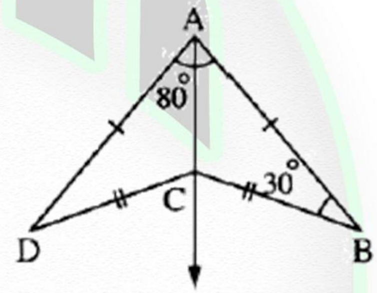
....

In each of the following figures , find the required under each figure :

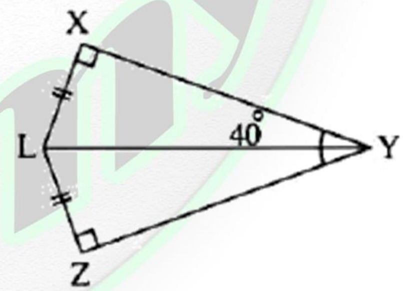
9) $\overline{AB} \cap \overline{CD} = \{M\}$ m (∠ D) = ……… °



10) m (∠ D) = ……… ° , m (∠ BAC) = ……… °



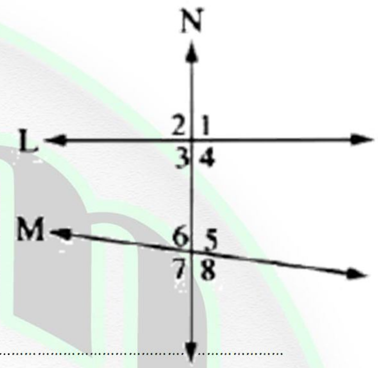
11) m (∠ XLY) = ……… °



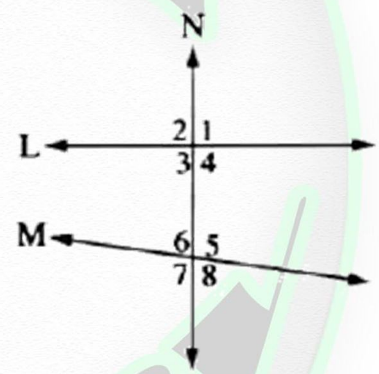
4-5 Parallelism

Angles formed from two straight lines and a transversal :

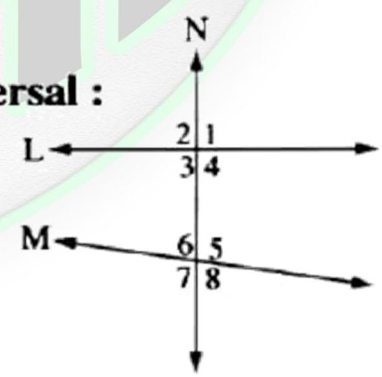
❶ Pairs of alternate angles :



❷ Pairs of corresponding angles :



❸ Pairs of interior angles on the same side of the transversal :

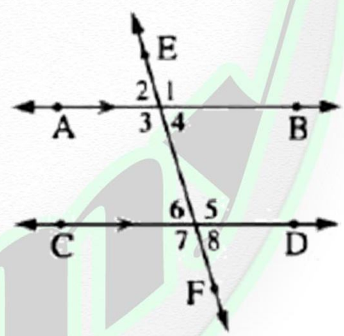


Relation between pairs of angles formed from two parallel straight lines and a transversal to them :

If a straight line intersects two parallel straight lines , then each two alternate angles are equal in measure.

If a straight line intersects two parallel straight lines , then each two corresponding angles are equal in measure.

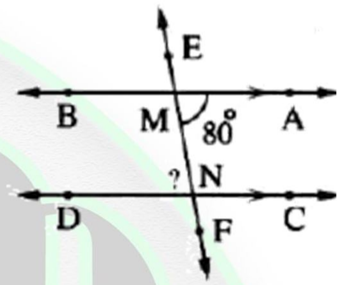
If a straight line intersects two parallel straight lines , then each two interior angles in the same side of the transversal are supplementary.



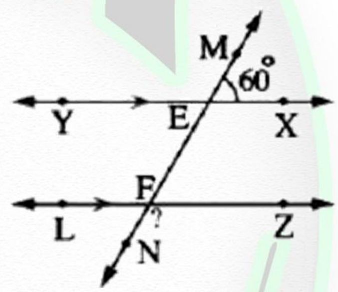
Example:

In each of the following figures , find the measure of the angle which is marked by " ? " giving reason.

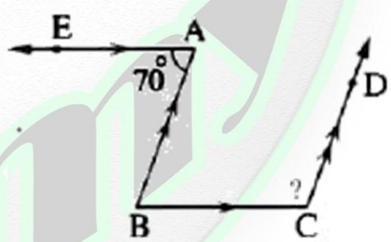
1)



2)



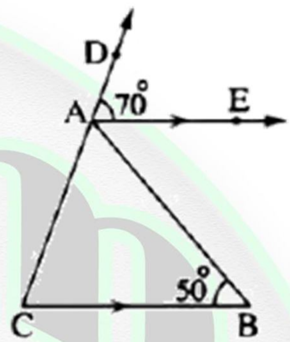
3)



Example:

4) **In the opposite figure : $\overrightarrow{AE} \parallel \overrightarrow{BC}$, $D \in \overrightarrow{CA}$, m ($\angle$ DAE) = 70° and m ($\angle$ B) = 50° Find giving reason :**

1 m ($\angle$ EAB) **2** m ($\angle$ C) **3** m ($\angle$ EAC)



....

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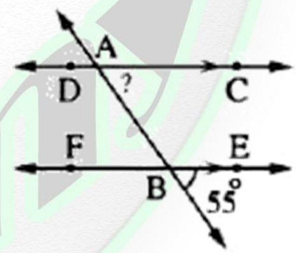
....

In each of the following figures , find the measure of the angle which is written under each figure (giving reason) :

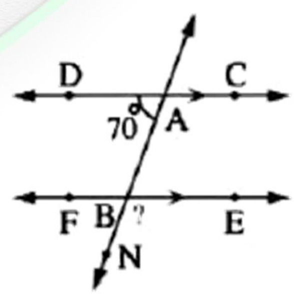
5) m ($\angle$ CAB) = ... °

The reason is ....



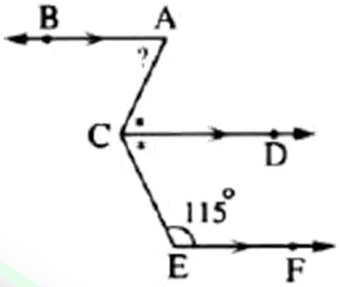
6) m ($\angle$ EBN) = ... °

The reason is ....



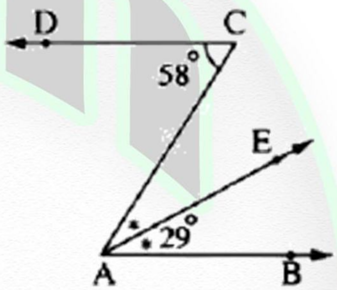
7) m (∠ A) = ……… °

The reason is ………

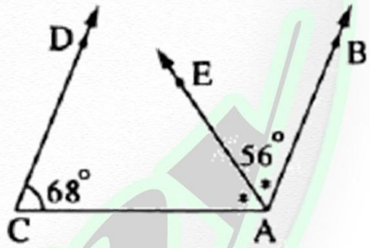


EXAMPLE: **In each of the following , show why $\overleftrightarrow{AB}$ is parallel to $\overleftrightarrow{CD}$**

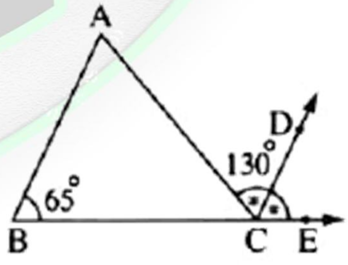
8) ……………………………………………………



9) ……………………………………………………



10) ……………………………………………………



From the previous, we deduce that :

If parallel straight lines divide a straight line into segments of equal lengths , then they divide any other straight line into segments of equal lengths.

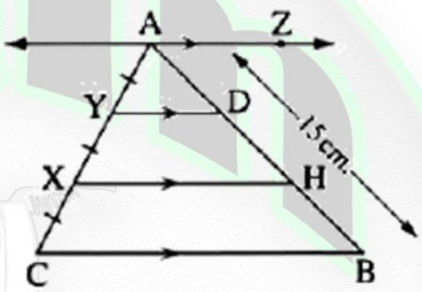
Example:

11) **In the opposite figure :**

$\overleftrightarrow{AZ} \parallel \overline{YD} \parallel \overline{XH} \parallel \overline{CB}$

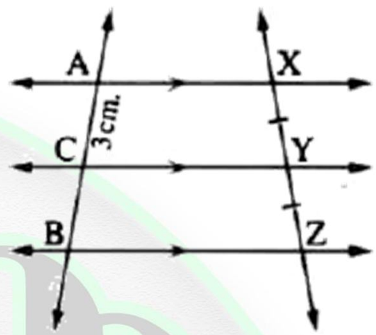
, AY = YX = XC and AB = 15 cm.

Find the length of $\overline{BD}$ showing the reason.



Complete under each figure of the following figures :

12) AB = ... cm.



13) BH = ... cm.

The perimeter of Δ ADY = ... cm.